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THE ROYAL AERONAUTICAL SOCIETY

JULY 1953

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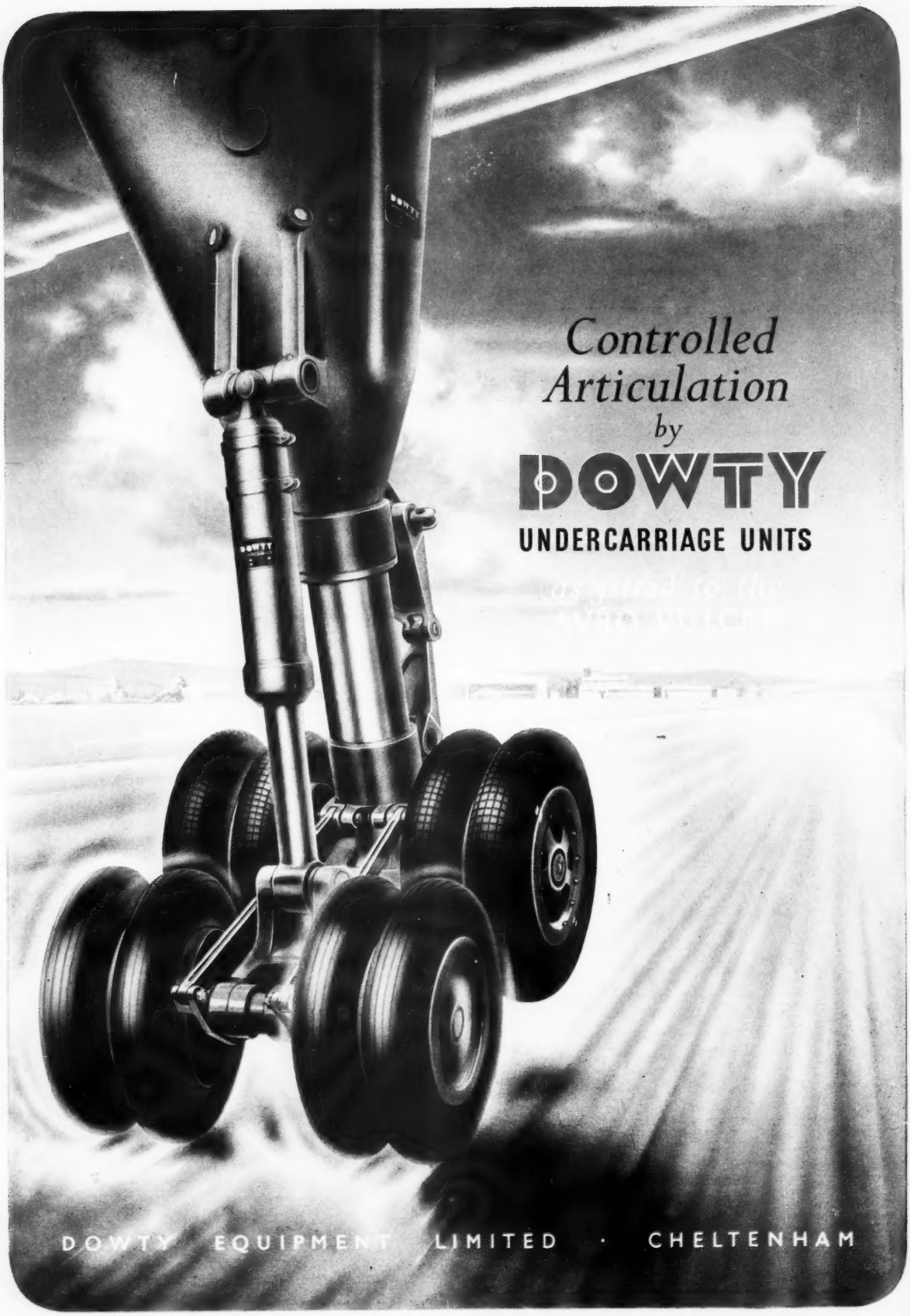
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Journal of
THE ROYAL AERONAUTICAL SOCIETY

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All communications for publication in, and advertisements for, the Journal should be addressed to the Editor, Joan Bradbrooke, A.R.Ae.S.; all communications on general matters affecting the Society should be addressed to the Secretary, A. M. Ballantyne, T.D., B.Sc., Ph.D., A.F.R.Ae.S.

None of the papers or paragraphs must be taken as expressing the opinion of the Council of the Royal Aeronautical Society unless stated otherwise.

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The Royal Aeronautical Society

(Founded 1866)

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HER MAJESTY THE QUEEN

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*In the Coronation Honours List, June 1953, a Knighthood was conferred on Mr. S. Camm, and Mr. E. T. Jones received the C.B.

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NOTICES

An annual sum of £250 is available for premium awards, usually 15 guineas each, for papers published in the Journal. Members and non-members of the Society are invited to submit papers on any aspect of aeronautics

Loyal Address

A Loyal Address was sent on behalf of the Members of the Society to Her Majesty the Queen on the occasion of Her Coronation on 2nd June 1953.

August Bank Holiday

The Library and Offices of the Society will be closed from 5 p.m. on Friday 31st July until 9 a.m. on Tuesday 4th August 1953.

Election of Vice-Presidents

The following Members of Council were elected Vice-Presidents of the Society for the year 1953/1954:

Mr. S. Camm, C.B.E., F.R.Ae.S.
Mr. G. R. Edwards, C.B.E., B.Sc., F.R.Ae.S.
Mr. P. G. Masefield, M.A., F.I.A.S., F.R.Ae.S.

Anglo-American Conference, London, September 15th-17th 1953

Full particulars, including the list of papers to be read, were sent to all members with the May Journal. Will those members who wish to register as delegates please return the completed entry form by 31st July 1953 which is the closing date for all entries.

Associate Fellowship Examination

Home entries for the December 1953 Fellowship Examination should be received by the Secretary not later than the 31st August 1953. The lists for entries outside the United Kingdom have been closed.

Coronation Honours

Among the members of the Society who were honoured by The Queen on the occasion of Her Majesty's Coronation were the following:

Knights Bachelor

S. Camm, C.B.E. (*Fellow*)
T. O. M. Sopwith, C.B.E. (*Honorary Fellow*)
W. R. Verdon Smith (*Companion*)

C.B.

E. T. Jones, O.B.E., M.Eng. (*Fellow*)

Order of St. Michael and St. George

R. E. Hardingham, O.B.E. (*Fellow*)

K.B.E.

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A. Fage, F.R.S., A.R.C.Sc. (*Fellow*)

O.B.E.

G. B. Bolt (*Associate Fellow*)
H. J. Curnow, B.Sc. (*Associate Fellow*)
C. T. Wilkins (*Fellow*)

Awards to Members

Captain J. L. Pritchard, C.B.E., *Honorary Fellow*, Mr. C. G. Grey, *Founder Member* and *Honorary Companion*, and Mr. A. F. Houlberg, *Associate Fellow*, have been awarded the Paul Tissandier Diploma by the Fédération Aéronautique Internationale for their "Services to Aviation."

Miss Elizabeth G. McGill (*Fellow*) received the 1953 Award for meritorious contribution to engineering from the Society of Women Engineers at their recent National Convention in New York.

Mr. G. A. V. Tyson (*Associate Fellow*) has been awarded the Brackley Memorial Trophy by the Guild of Air Pilots, in recognition of his work in connection with the "Princess" Flying Boat.

Members' New Appointments

Squadron Leader W. H. Brooks (*Associate Fellow*), formerly Ministry of Supply, has recently joined Aircraft Materials Ltd., London, N.W.1, as Technical Representative.

F. J. Edwards (*Associate Fellow*), formerly Experimental Officer of the National Gas Turbine Establishment, has been transferred to Engineer III at the Directorate of Aircraft Production.

G. W. H. Gardner, C.B.E. (*Fellow*) has been appointed Director General of Technical Development (Air) at the Ministry of Supply.

A. H. C. Greenwood (*Associate*) has recently been appointed Service Manager (Weybridge Works) and Technical Assistant to the General Manager, Aircraft Division, Vickers-Armstrongs Ltd.

W. M. Hargreaves, C.B.E. (*Associate Fellow*) has been appointed Northern Divisional Controller at the Ministry of Civil Aviation's Headquarters at Liverpool.

D. Houghton (*Associate Fellow*) has joined the staff of the College of Aeronautics as a Senior Lecturer.

Lt.-Cdr. (E) H. A. Jarrett, M.B.E., R.N. (*Associate Fellow*), who was recently placed on the Retired List, has joined the Ministry of Supply and has been appointed to a post at Vickers-Armstrongs (Supermarine), Winchester.

Group Captain E. L. Mole, B.Sc. (*Associate Fellow*), formerly Director of Civil Aviation in the Bahamas, is now established as an aviation consultant in this country.

L. F. Nicholson (*Associate Fellow*) has been appointed Head of the Aerodynamics Department at the Royal Aircraft Establishment, Farnborough. He was formerly Head of the Supersonic Division of the Department.

G. M. Stallibrass, O.B.E. (*Associate*) has been appointed Director of Navigational Services (Aerodromes), Ministry of Civil Aviation.

D. L. Taylor (*Associate Fellow*), previously a Mechanical Test Engineer with Saunders-Roe Limited, has taken up an appointment with the Lockheed Aircraft Corporation, Burbank, California.

Elliott Memorial Prize

The Elliott Memorial Prize has been awarded to: Aircraft Apprentice A. Mallen of the September 1950 Entry, who obtained the highest marks in the General Studies examination.

Graduates' and Students' Section—Visits

An all-day visit has been arranged to the **U.S.A.F Station at Withersfield, Essex, on Saturday 18th July 1953.** Visitors will be able to inspect the Republic F-84 aircraft and associated equipment, together with the maintenance and operational control facilities. Lunch will be provided and there will be transport from Braintree Station connecting with the 10.40 a.m. train from Liverpool Street. If numbers permit, the reduced return fare will be 10s.

Numbers are strictly limited, and application must be made to the Honorary Visits Secretary of the Section, C. B. Redgate, *Graduate*, 83 Wandle Road, Morden, Surrey, **before 8th July.**

Money should not be sent with applications, but members are asked to state whether they intend to travel by train or private car.

An all-day visit has also been arranged to the **Royal Aircraft Establishment, Farnborough, on Wednesday 26th August 1953,** from 10.30 a.m. to 4.30 p.m. Lunch will be available. Visitors will be able to see the wind tunnels, structural and engine test rigs, and the flight test department.

Applications should be sent to the Honorary Visits Secretary, C. B. Redgate, 83 Wandle Road, Morden, Surrey, before **10th July 1953,** and members are asked to state their grade of membership of the Society and that they are British Subjects.

Elections

The following is a list of new members and transfers of Membership of the Society:

Associate Fellows

Roland Shirley Austin (*from Associate*)
Charles Reginald Burgess (*from Associate*)
Denys Martin Callow
Albert Frenkel (*from Companion*)
Reginald Charles Grinter
Gerald Langer
Leslie Raymond Nash (*ex-Graduate*)
Robert Derek Samson (*from Associate*)

Associates

Ronald Alfred O'Leary Cashel (*ex-Student*)
Donald Paginton
Eric Mark Patience

Graduates

Bryce Robert Bollert
Jack Cohen
Alan Grenfell Cole
David Charles Cornish
Lewis John Fennell
Michal Jan Henryk Giedroyc
Anthony John Heath (*from Student*)
Gilroy Reginald Ketley (*from Student*)
Alastair John Macdonald
Terence Graham Morton
Basil Kynvin Arthur Nixon
Geoffrey Arthur Clayden Searle (*from Student*)
Patricia June Wilson (*from Student*)

Students

Kevern Barbeary
Donald Clive Clayton
Denzil De Villiers
Christopher John Hawkins
James Albert Longley
Sivaramannair Kesavan Nair
Clive Russell
Peter Shu-Yan Wong
Michael Leyland Woodward

Companion

Waclaw Socha

Acknowledgments

The Council acknowledge with grateful thanks the return of back numbers of the Journal from A. Rose Esq., *Associate Fellow.*

London

September 14th

Forty-first Wilbur Wright Memorial Lecture—Structures. Professor N. J. Hoff. Royal Institution, 21 Albemarle Street, W.1. 6 p.m. Members may obtain tickets for visitors from the Secretary.

September 15th-17th

Fourth Anglo-American Aeronautical Conference. University College, London. (Delegates only.)

Branches

July 6th

Halton—Film Night. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

July 13th

Halton—Civil Aircraft Maintenance Engineering. To be arranged by courtesy of the Society of Licensed Aircraft Engineers. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

July 20th

Halton—Junior Members' Night. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

Journal Binding

Self-Binder Cases

Self-Binder cases of the "Easibind" type are available from the offices of the Society. These binders are for members who do not have their Journals permanently bound, or who wish to keep their Journals together during the year for binding later.

These cases will hold 12 Journals which are kept in place by means of flexible steel wires. Journals can be inserted or withdrawn easily without damage, so preserving the contents for permanent binding later. The Journals will open flat at any page.

The binder is strongly made in durable dark blue leather cloth on stiff board covers and has gold lettering on the spine. The year is not blocked on the spine but there is a panel on which members who wish to use the binder as a permanent case can put the date.

The cost is 11s. 6d. each including postage and packing for either the size to fit 1952 and previous Journals, or for the size to fit the Journal from January 1953, which has been increased in size. Orders and remittances should be sent direct to the Secretary at the Offices of the Society and it is *important to state whether the old size or new size is required.*

Permanent Binding

There is no increase in the price of permanent binding of Journals. The prices are:—

1952 Volume (including packing and postage) 16s. 0d.
Previous Volumes (including packing and postage) 18s. 0d.

Journals, *with a note of the name and address of the sender,* should be sent direct to the Lewes Press, Friars Walk, Lewes, Sussex, and the remittance to the Secretary at the Offices of the Society.

Changes of Address

To assist in keeping the records of members correct and up to date the Secretary will be glad if all members will notify him as soon as possible of changes of address.

When notifying changes please give the following particulars:—

Name (in block letters).
Grade of membership.
New address (in block letters).
Old address.

Changes of address must be received *before the 15th of the month* in order to be effective for the following month.

Journal of The Royal Aeronautical Society

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS

VOLUME 57 JULY 1953 NUMBER 511

Sixth Louis Bleriot Lecture

Making Commercial Aircraft Pay

by

Ingénieur Général G. Du MERLE, F.R.Ae.S.

The Sixth Louis Bleriot Lecture was given in London by General Guy du Merle on 19th March 1953 at the Institution of Mechanical Engineers, Storey's Gate, S.W.1. Mr. G. H. Dowty, F.R.Ae.S., President of the Society, presided and welcomed particularly M. Jules Jarry, President of the Association Française des Ingénieurs et Techniciens de l'Aéronautique, and members of A.F.I.T.A., who had come over from France for the occasion.

Introducing the Lecturer, the President outlined his career. After leaving school General du Merle went into the corps of aeronautical engineers, which was then of civil status but had since been militarised. He spent two years at the Ecole Nationale Supérieure de l'Aéronautique, graduating in 1932, and obtained his certificate as a pilot of military aircraft including seaplanes. For the next two years he served as an engineer pilot at the Centre d'Essais en Vol (Flight Testing Centre) testing prototype aeroplanes at Villacoublay, during which time he flew both French and foreign aircraft. He next became Engineer at the Technical Department of Aeronautics at the Air Ministry and then Director of the Department. While a professor at the Ecole Nationale Supérieure de l'Aéronautique he had published a book on the construction of aeroplanes, a copy of which was in the Library of the Society. Upon being attached to civil aviation at the Ministry of Public Works and Transport, he founded and directed for two years the National School of Civil Aviation at Orly. General du Merle was then appointed Director of Navigation and Air Transport at the same Ministry, and represented France at a number of international meetings on Civil Aviation. Appointed Ingénieur Général de l'Air, he had regained for a short time his original Ministry, which he now represented on the Interim Committee of E.D.C. He had been on missions to England a number of times since the end of the war, and in 1947 had flown several British aeroplanes, including the Sea Fury, the Meteor and the Vampire. He was a Fellow of the Royal Aeronautical Society, and was also an Officer of the Lêgion d'Honneur and held the Medaille de l'Aéronautique. General du Merle was at present Commandant des Ecoles et Stages at the Ministère des Travaux Publics et des Transports (Officer in charge of Schools and Courses).

Foreword

Aviation, the infant prodigy of the age, the daredevil, and the idealist, has in the past few years (and especially since the Second World War) come into its "Economic Age."

One wonders what Louis Bleriot, whose memory we are celebrating today, would have said about this metamorphosis.

If we think of Bleriot as an idealistic pioneer for whom aviation was a kind of super-deity, we might suppose that the economic approach would have seemed to him like "clay hands on his marble idol." But Louis Bleriot was more than an inspired visionary, he was a man of wisdom with an eye for the future. During his own lifetime he saw the "flying machine" move from the sporting ground to the battlefield—bearing evil fruits along with good. His particular contribution, overseas flight, linked the great nations of the world in a degree of closeness previously considered impossible. The magnitude of this event is especially pertinent to our two countries because of the link it forged across the Channel.

The pioneer after whom this lecture is named was personally and powerfully concerned with the peaceful and prodigious expansion of the industry which he so effectively helped to build up from nothing. This spirit of progress and usefulness is further attested by the work of one of Bleriot's contemporaries, M. Louis Bréguet, a man intensely concerned with the study of aircraft economics.

Few of the pioneers of aviation will quarrel with its progressive evolution. The golden anniversary of the first flight of the Wright Brothers offers a convenient landmark from which to survey a half-century's continuous, rapid and fruitful progress, and to train our eye on the future.

Aviation in its new-found maturity presents the highest challenge to the combined efforts of the

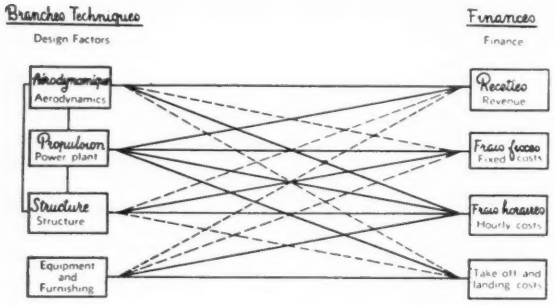

FIGURE 1.

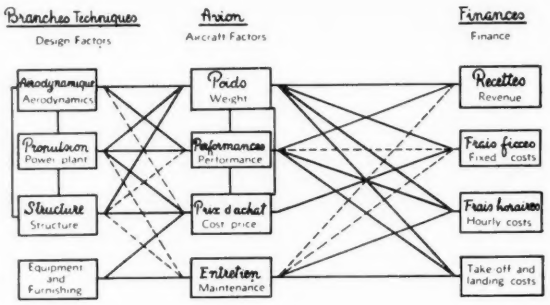

FIGURE 2.

economist and the idealist. It is a vital and dynamic field, one demanding the utmost in man's powers of initiative and resourcefulness in order to harness the capabilities shown in wartime to peaceful commercial pursuits. It is a field pregnant with benefits for all mankind; and it is big business.

May atomic energy soon be prompted to follow its example !

1. Introduction

Any survey of economic prospects involves a comparison of expenditure with revenue. It is therefore appropriate to consider in detail a few of the principles which control the scales on the balance in our particular field.

Considered as a whole, commercial air transport revenues are firstly restricted by the financial limitations of the customer.

Revenues are essentially the product of fares by the number of passenger-miles or by the ton-miles flown. These two factors vary, as always, in inverse proportion: that is, if the price is high the traffic is low. And their interaction, which determines the scope of the potential market, is extremely sensitive. At least in Europe, fares are still above the desired level if commercial aviation must cease to deserve its wag's definition: a charity organisation designed to permit the rich to travel rapidly and cheaply. In short, since the condition precedent to greater airline traffic is lower fares, it follows that any endeavour towards improving the economic outlook for commercial aviation must have as its aim reduced passenger and freight fares.

For any given airline, revenues are further reduced by competition. This is especially in evidence on the great international routes. This particular point will be dealt with later.

Expenses are roughly distributed as follows for an airline operating a variety of routes ranging from the local short-hauls in Europe to the international long distance services, assuming that it maintains its own fleet and has commercial agencies throughout the world:—

	Per cent.
Overheads and financial costs ...	13
Commercial costs (sales)	12
Staging costs (technical and commercial)	15
Hourly operating costs (fuel 17, air crews 10, passenger service 3)	30
Shops	30
Total	100

To clear the matter, it would be useful to seek for an apportionment of these costs enabling us to determine the share related to a specific type of aircraft and given phases of its operation. Unfortunately this is not easy, for this analysis is not common in the accounts of airline operators and this will lead us to make some arbitrary cuts in some headings which are usually grouped, such as staging and shop costs. Nevertheless the following distribution into four classes can be suggested:

(*i*) Overhead expenses or indirect costs.
These cover the administrative operation of the transport airlines as well as the operation of their commercial and technical installations throughout the world; they remain constant regardless of the particular type of aircraft operated (about 35 to 40 per cent.).

(*ii*) Yearly fixed costs.
These expenses are computed for the entire fleet of aircraft, but are independent of the number of hours flown (*e.g.* amortisation, insurance, and so on) (about 8 to 12 per cent.).

(*iii*) Hourly operating costs.
These are proportional to the number of hours flown (*e.g.* fuel, lubricant, air crew, maintenance, and so on) (about 50 to 55 per cent.).

(*iv*) Staging costs.
Here we shall consider only those expenses directly connected with the aircraft itself (*e.g.* landing and parking fees, tyre wear, extra fuel consumption during climbs and descents, as compared with cruising consumption, extra maintenance at staging point, and so on). "On the ground" supervision of passengers and

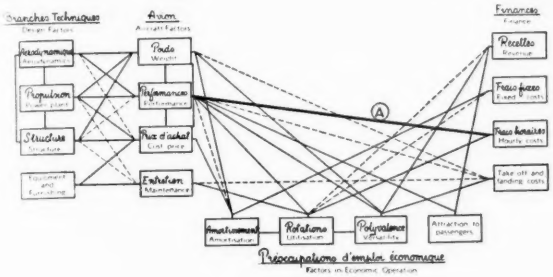

FIGURE 3.

cargo and all other commercial transactions at the staging point are related to traffic itself, whatever may be the equipment used for this purpose, and for that reason are accounted for under the title "overheads" already mentioned. There remains about 2 per cent.

The last three classes make up direct operating costs. These are the only costs we shall consider and it is essential to point out that they make up about 60 to 65 per cent. of the total.

We shall not here go any further into details of cost distribution between the items. Well-proven methods are being applied to this problem in various countries and with resulting differences that it is not our intention to develop. We are neither concerned with obtaining numerical results nor with establishing a prize-list classification of present and future aircraft. It is sufficient here that the principle of such a distribution be admitted before pointing out the angle from which we propose to deal more thoroughly with the subject.

We do not intend to investigate either civil aviation as a whole, or the working of an air transport enterprise. Rather, we shall deal with the aircraft themselves, discarding all consideration of commercial and administrative overhead expenses and focusing on the direct operating expenses and revenue. Our task is to transfer into the technical field the requirements aiming at profitable operation, such requirements being appropriate to the airline operators' choice—and to the design —of future aircraft.

Thus defined, the problem is nevertheless too intricate to be studied profitably until a method of approach is selected.

In point of fact, the design of commercial aircraft merges numerous specific techniques, but these may roughly be classified as follows: aerodynamics, propulsion, structural design and, lastly, equipment and installation, in which we shall include construction details intended to facilitate operation and maintenance.

Each of these four branches plays an important part in the profit earning capacity of the aircraft. Then, if we try to show the connections linking them to the key factors of earning capacity (*e.g.* revenue, fixed costs, hourly costs, take-off and landing costs) by means of a symbolic diagram, such as Fig. 1, the best we can do is to plot direct, or important, effects with a solid line and indirect or subsidiary ones, using a dotted line.

Some kind of cipher-grid should then be used to decipher this obscure, but nevertheless significant data. The first factor to come to the mind (and it is certainly an indispensible one), is the aircraft itself: its performance, its cost, its maintenance and its weight; this last item should represent not only the all-up weight but also its components, with special emphasis on the variable load, fuel load plus payload. The characteristics of performance, cost, and so on, are, of course, inter-related and react one upon the other. Since they are linked, on the one hand, to the various technical branches and, on the other hand, to the various items of profit-earning capacity, they resemble a selector-filter which enables us to interpret the network of connections binding the constructor's technical force at one end to the airline operators' financial staff at the other. This is the approach Mr. Peter Masefield[1] used for his outstanding lecture in October 1948, his work being one of the most important fundamentals on civil aviation published up to the present time.

It is possible to superimpose upon this chart another grid reflecting some of the prime considerations of economical operation, either explicit or latent in the minds of those persons responsible for the selection of an aeroplane. Some such factors are: utilisation, versatility, amortisation and attractiveness, the characteristic which describes the aircraft's ability to attract passengers.

This process is by no means a simplification of the problem, since it increases the number of connections instead of reducing it. Neither does it constitute a complete, consistent and self-sufficient system of analysis. This is why it was introduced into the symbolic diagram (Fig. 3) as a mirror intended to give a side-view of such diagram and not as a kind of necessary filter. In fact it reflects some connections poorly. These, therefore, must subsist in a shunt. It is obvious, for instance, that performance bears far more significantly upon hourly costs than is indicated by utilisation alone. Whence the direct connection indicated by "A," which is probably the most important one of the whole problem. Since, however, it has been dealt with many times, and in an extremely able manner (particularly in Mr. Masefield's paper), we shall refer to it now only summarily.

2. Weight* and Performance

It is appropriate to start with a rough outline of the all-up weight problem in its relation to payload in order to emphasise the paramount importance of lightness of structure. In point of fact, this principle merits the greatest effort towards new and better structural solutions, as well as towards the development of materials of high tensile strength.

2.1. WEIGHT ESTIMATION

The huge payloads carried by aircraft have become so commonplace that it is easy to assume that a few score of pounds (or even a few hundredweight on heavy

*Note that the conversion factor from tonnes (metric tons) to long tons (2,240 lb.) is 0·9842.

aircraft) more or less in the structural weight or in the engines and various accessories are of slight consequence. Some people are tempted to laugh when they find that on the drawing board aircraft weight data are computed down to fractions of an ounce for each specific item. But let us keep in mind that accounting personnel in the business offices of the airlines do the same thing. The accumulation of useless ounces runs into hundredweights, and even tons, with remarkable alacrity.

It would be serious enough if neglect of this capital point only had the effect of lowering magnificent technical successes to the level of "fair designs" (and unfortunately this happens very frequently); but it is our purpose to point out that this may even result in unusable aircraft, especially in the arduous field of the very long range aircraft.

Let us try to express as simply as possible the all-up weight of an aircraft of that class. When assuming, for simplification, that the fraction of the all-up weight which corresponds to the empty weight (in which we shall include the crew) depends only upon structural design of the aircraft and remains unchanged in a given programme when the aircraft size varies, then we may call "constructional efficiency" η_c the remaining fraction. Thus we have

$$\eta_c = \frac{F+G}{W_{to}}$$

where F is the weight of fuel and oil burnt, G a constant representing the programme design payload and W_{to} the maximum all-up weight of the aircraft.

Let us try to compute G, firstly in the case of a propeller aircraft. If we call P_c the engine cruising power, and c the corresponding specific consumption, we have, as the aircraft progresses on its route:

$$dF = -cP_c dt$$

where t is the time. Let us assume first, to simplify, that the flight is made with zero wind—we shall enlarge later upon the case of a headwind. We have then:

$$dt = \frac{dD}{V}$$

where D is the distance flown and V the true air speed (or ground speed since there is no wind).

Then $$dF = -c\frac{P_c}{V}dD .$$

But, calling η_p the propeller efficiency, we have

$$V = \frac{C_L}{C_D}\frac{\eta_p P_c}{W}$$

and hence $$dF = -\frac{c}{\eta_p}\frac{C_D}{C_L}W dD .$$

But $$dF = dW$$

and so $$\frac{dW}{W} = -\frac{c}{\eta_p}\frac{C_D}{C_L}dD .$$

If we assume that the flight is made at a constant angle of incidence, and hence at a constant C_L/C_D ratio—and that the propeller efficiency η_p can be likewise maintained at a constant value, the integration is obvious and gives:

$$W_1 = W_{to}\exp\left\{-\frac{c}{\eta_p}\frac{C_D}{C_L}R\right\}$$

calling W_{to} the take-off weight and W_1 the landing weight when a given range R has been flown. To simplify, we shall neglect the differences in consumption and speed due to climb and descent (we may do that for a very long range) and we shall assume that the whole of the flight is made in cruising conditions. We have then

$$F = W_{to} - W_1 = W_{to}\left[1-\exp\left\{-\frac{c}{\eta_p}\frac{C_D}{C_L}R\right\}\right]$$

$$\eta_c W_{to} = W_{to}\left[1-\exp\left\{-\frac{c}{\eta_p}\frac{C_D}{C_L}R\right\}\right]+G ,$$

and finally

$$W_{to} = \frac{G}{\eta_c - 1 + \exp\left\{-\frac{c}{\eta_p}\frac{C_D}{C_L}R\right\}} \quad . \qquad (1)$$

In order to simplify, fuel reserves have not been accounted for, but they can be computed in each particular case as a function of the distance flown and come in by deduction from R.

Equation (1) shows in the first place that, with the assumptions made, there is an absolute limit to the possible range to be considered, because the weight tends to infinity when the denominator cancels out, *i.e.* when R approaches the critical value given by

$$R = -2{\cdot}3\frac{\eta_p}{c}\frac{C_L}{C_D}\log(1-\eta_c) \quad . \quad . \qquad (2)$$

where we find, with a slightly different appearance, a formula evidenced very long ago by M. Louis Bréguet.

Incidentally, according to our assumptions, this limit is independent of the payload G, which only governs the take-off weight value. It depends only upon the level of the technique in use, *i.e.* upon the values of

$$\eta_p \cdot \frac{C_L}{C_D} \cdot \quad c \quad \text{and} \quad \eta_c$$

available at the particular moment in airframe and/or engine engineering.

But equation (1) shows moreover that, once the programme is issued, if it comprises rather stringent conditions, *i.e.* if the adopted maximum range R approaches closely the critical value, the choice of too heavy a design bringing down η_c below the value

$$1-\exp\left\{-\frac{c}{\eta_p}\frac{C_D}{C_L}R\right\}$$

which is necessarily rather close by, can irremediably jeopardise the development and give, whatever the all-up weight may be, an unusable aircraft on the given range.

For jet aircraft, a similar calculation gives, with the same basic assumptions, the following formulae

$$W_{to} = \frac{G}{\eta_c - 1 + \exp\left\{-\frac{\xi}{V}\frac{C_D}{C_L}R\right\}} \quad . \qquad (3)$$

$$R = -2{\cdot}3\frac{V}{\xi}\frac{C_L}{C_D}\log(1-\eta_c) \quad . \quad . \qquad (4)$$

where ξ is the specific consumption of jet engines in

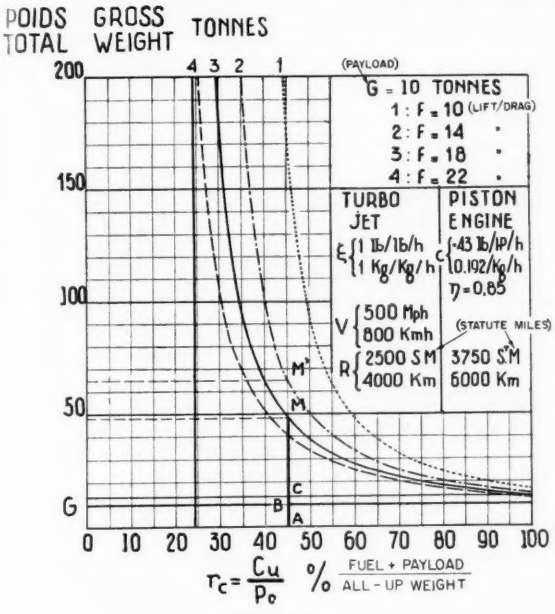

FIGURE 4.

cruising conditions, and which—*mutatis mutandis*—calls for comments quite similar to those already given*.

Figure 4 shows the all-up weight as a function of η_c with the four following values given to lift/drag ratio: 10, 14, 18 and 22, and with other parameters having values that are probable at the present day.

By a numerical and purely coincidental occurence, but nevertheless a very convenient one, this diagram may be used for piston-engined aircraft on a stage length of 3,700 miles, as well as for jet aircraft with a cruising air speed of 500 m.p.h. on a stage length of 2,500 miles, for with the values assigned to the parameters (all these values corresponding to present technical possibilities) it is found that

$$\frac{\xi}{V} = 1{\cdot}5\frac{c}{\eta_p}.$$

Everything being equal, the maximum range of a jet aircraft and the maximum range of a piston-engined aircraft are then in the ratio of 1 to 1·5. In fact, on jet aircraft the values of the lift/drag ratio and particularly that of the constructional efficiency are higher owing to

*The Bréguet Aircraft Co. has suggested, from formula (4), the following simplified expression for the maximum range of a jet aircraft:—

$$R = \frac{12}{\xi}\sqrt{\left[\frac{Ka_0T_0'\lambda'}{S(C_D)^2}\right]}\ \log\left[\frac{1}{1-\frac{F}{W_{to}}}\right]$$

where K is the aerodynamical efficiency of the air intake, a_0 the value, for the minimum of the thrust curve, of the T_0'/T_0 ratio (equivalent ground thrust/maximum thrust at test-bed), λ' the aerodynamical aspect ratio of the wing, S the wing surface and F the fuel weight. The weights are to be expressed in kilogrammes and the wing surface in square metres.

The proportionality to $\sqrt{T_0}$ carries this remarkable consequence that any increase in test-bed thrust or in the number of jet engines gives a corresponding increase in range or decrease in fuel consumption.

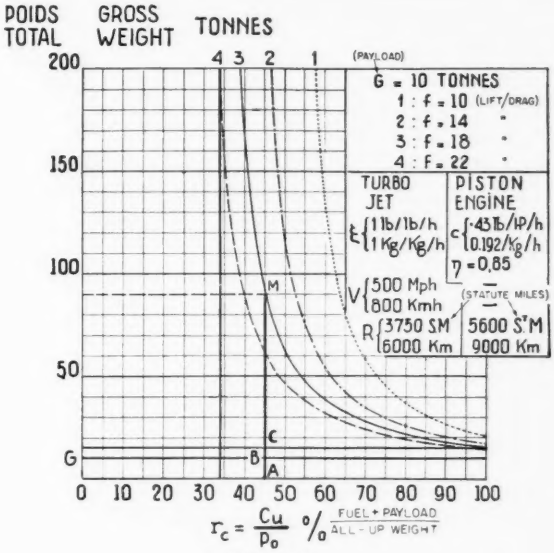

FIGURE 5.

the lower unit weight of the jet engines and this slightly diminishes the difference. From this result we felt justified in formulating a separate diagram (Fig. 5) (still rather futuristic) for jet aircraft for the 3,700 miles no-wind stage which was shown in Fig. 4 for piston-engined aircraft. Obviously this diagram may also be used for these latter aircraft on a 5,600 mile stage.

The "marginal" character of the 3,700-5,600 miles programme (Fig. 5) is emphasised by the fact that, with a 45 per cent. design efficiency and a lift/drag ratio of 18, the aircraft cannot overcome an average headwind speed of 50 m.p.h. on this stage distance; Curve 2 would then lead to a prohibitive weight.

For a given curve (*e.g.* Curve 3) and for a given value of η_c (*e.g.* $\eta_c = 45$ per cent.) the all-up weight AM is composed of: AB which is the payload; BC the weight of fuel provided to carry the payload (C is the ordinate of the efficiency point $r_c = 1$); and CM the aircraft weight including the weight of fuel required to sustain it in flight.

The slope of hyperbolas in the vicinity of optimum η_c values currently obtained (in the order of 45 per cent. with reciprocating engines, and nearing 50 per cent. with jet engines) shows rather eloquently the decisive effect of η_c, especially when drag/lift ratios actually used are not the optimum.

This may happen even with an aerodynamically refined aircraft. Headwind can be taken into account by introducing into formulae (1) and (2) a fictitious value of the product $\eta_p C_L/C_D$, this being less than the actually used value. In fact, it is $P_c/(V-v)$ rather than P_c/V which is to be introduced when computing dF. A simple way to do this is to subtract from $\eta_p C_L/C_D$ a value equal to Wv/P_c. For 50 m.p.h. (or 45 knots) winds and with current values of the various parameters, this reduction produces a variation of about 4 points in the lift/drag ratio, *i.e.* it nearly corresponds to moving down one curve in Figs. 4 and 5.

The critical character of the 3,700-5,600 miles programme (Fig. 5) is emphasised by the fact that, with a 45 per cent. constructional efficiency and a C_L/C_D ratio of 18, the aircraft cannot overcome an average headwind of 50 m.p.h. on this stage distance; Curve 2 would then lead to a prohibitive weight.

These summary remarks are outlined here to forewarn against excessive optimism when interpreting the curves of Figs. 4 and 5, and to point out that in spite of the high lift/drag ratio values obtained today, the weight data estimates are still far from simple matters. We are not yet clear of those diagram zones where a steep slope of the curve requires the utmost vigilance in order to prevent total failure.

Should we want to evidence the facts in another manner, we could say as well that, when referred to the optimum theoretical aircraft, an increase in the empty weight at the initial development stage results in an additional load to be added to the payload G, which normally should not be changed at this stage. At the final design stage, unfortunately, it has to be reduced !

As equations (1) and (3) show that W_{to} is proportional to G for any given values of other parameters, the effect of an excess in empty weight is immediately evidenced by its relative importance as compared with the payload G, and not as compared with the all-up weight. A structural weight increase of one ton, which represents 10 per cent. of G, will cause an increase of no less than 10 per cent. in the all-up weight, *i.e.* 6·2 tons in the case of Curve 3 of Fig. 5 at $\eta_p = 50$ per cent., and up to 11·5 tons for the same aircraft flying in the same conditions with a headwind of 50 m.p.h. (Curve 2 of the same figure).

2.2. COMMERCIAL SPEED

Throughout this paper, reference will constantly be made to commercial speed, which is the block-to-block average speed. This speed is obviously lower than the average cruising speed, even if allowance is made for wind, because of the dead times due to taxying, take-off, landing and corresponding procedures, and also due to climb and descent rates when the forward speed is different (always in climb and often in descent) from that of cruising.

Obviously, the shorter the flight distances, the greater will be the net effect of these dead times.

It is of interest to see, as in Fig. 6, a graphical representation of commercial speed versus the total distance flown and the number of stops. Obviously, some assumption must be made for the mean practical value of dead times. Available evidence shows, however, that these do not vary considerably with aircraft type. The assumption I made is as follows: —

> $\frac{1}{2}$-hour for the whole of ground manoeuvres and taxying, together with take-off and landing procedures, *i.e.* for the time elapsed before the moment when the aircraft in climb steers to its course and the moment when the aircraft in descent begins its approach at the altitude of about 5,000 ft.
>
> $\frac{1}{4}$-hour difference between the time actually elapsed in climb plus that in descent and the time which would have been necessary to cover the same distance at cruising speed (strictly speaking, this lag is affected by the wind, but a mean value has been assumed).

This $\frac{3}{4}$-hour will be repeated at each stop.

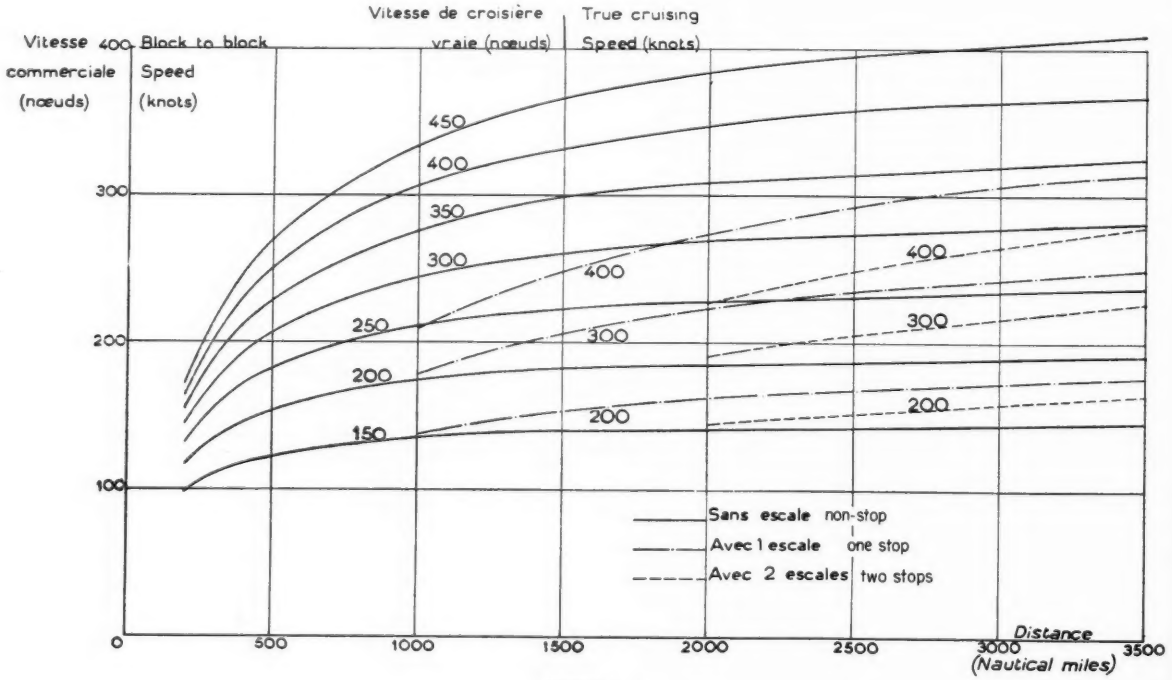

FIGURE 6.

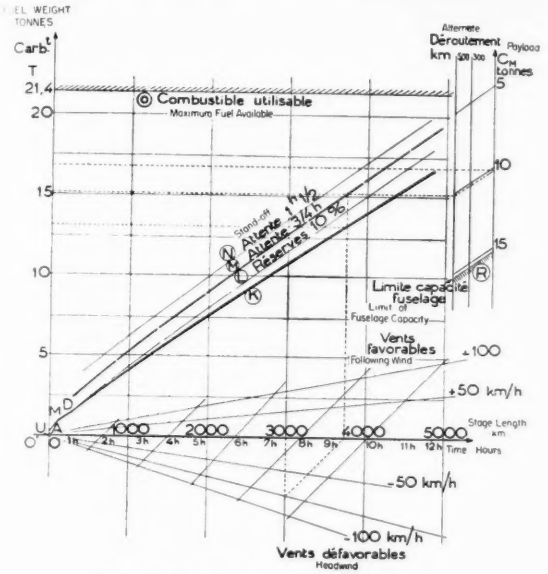

FIGURE 7. Armagnac S.E. 2010 (75 T.).

An attempt has also been made to show the effect of one or two stops (technical or commercial) on the mean commercial speed for the whole flight distance. For this purpose a mean stop duration of $\frac{3}{4}$-hour has been assumed.

Thus, Fig. 6 shows that, with the assumption made, an aircraft having a cruising air speed of 250 knots cannot obtain, even with zero wind, a higher commercial speed than 210 knots (242 m.p.h.) on a flight distance of 1,000 nautical miles.

On a flight distance of 3,000 nautical miles, with the wind zero, the cruising air speed of (A) a non-stop slow aircraft flying together with (B) a high speed stop-making aircraft which cruises at an air speed of 400 knots (460 m.p.h.), need not exceed (1) 334 knots (385 m.p.h.), or (2) 284 knots (326 m.p.h.) to keep pace with the high speed aircraft making (1) one stop, or (2) two stops, respectively. These differences decrease in case of a headwind, the effects of which are proportionally greater on the slow aircraft than on the high speed aircraft.

2.3. DETERMINATION OF PROFITABLE OPERATION ON A GIVEN ROUTE

Let us try and find a convenient representation of expenditure and revenue as a function of the stage length, with an assumption about headwind, which is one of the factors which closely link profitable operation to a given route.

I shall not reconsider the method of determining this assumption to correspond to the given stage. Mr. Peter Masefield has previously explained this matter at full length. His solution may be roughly summed up as follows: to obtain a 95 per cent. practical regularity, any computation of the fuel quantity to be carried (and therefore of the available payload) requires the adoption of a value for average wind which is equal to the strongest headwind component not exceeded at cruising altitude on 85 per cent. of occasions in the worst season for the stage under consideration. For capacity computation it is the maximum wind not exceeded on 95 per cent. of the same occasions.

The first question is: what payload is to be carried under these conditions as a function of the stage length?

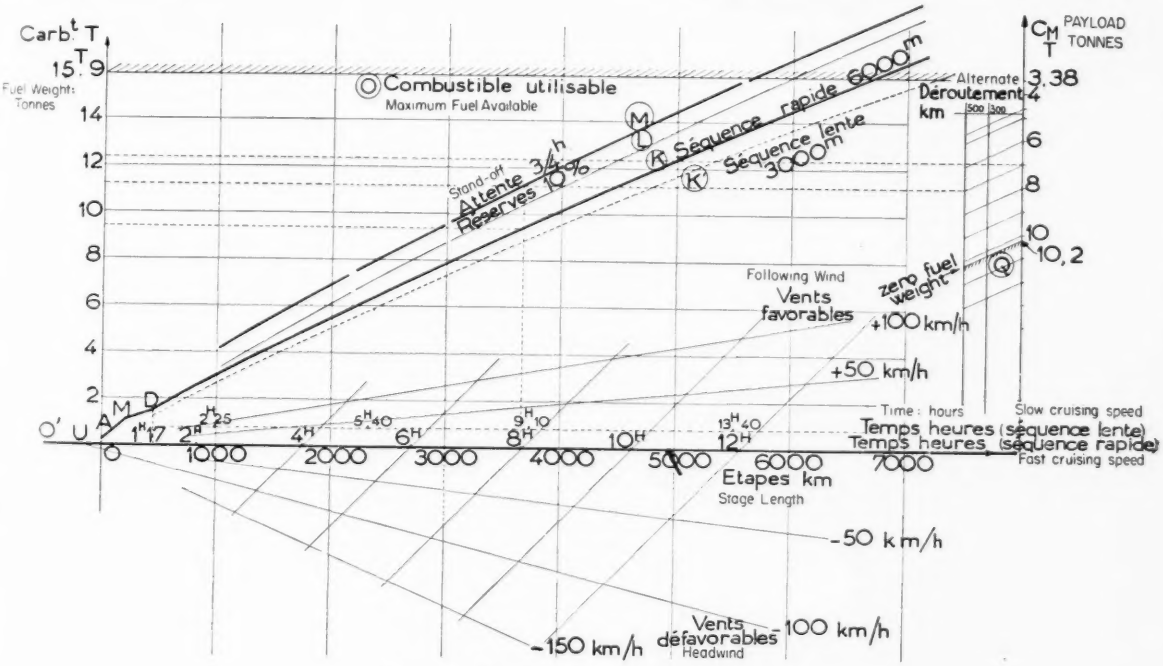

FIGURE 8. Constellation L.749 A (48 T. 535).

From this we can easily deduce the ton miles yielded by the aircraft in each particular case and consequently the revenue possibilities. The second question is: what is the commercial speed or "block-to-block" speed? Multiplying the corresponding elapsed time by the hourly costs, we arrive at the expenditure.

To determine these factors, we need a graph of the aircraft flight on which fuel load carried, fuel consumption, and commercial speed under various wind conditions can be readily seen.

A method devised by Ingenieur en Chef Kommer is commonly used by the French Secrétariat Général à l'Aviation Civile et Commerciale. This method is in substance roughly the equivalent of Mr. Peter Masefield's hour charts, but a different arrangement makes interpolation easier. The Kommer chart method appears with two applications for the French four-engined S.E. 2010 Armagnac and the Lockheed Constellation L. 749 A in Figs. 7 and 8 respectively.

Block-to-block flight times in hours are plotted on abscissae jointly with corresponding null-wind stage lengths, assuming that the true air speed at cruising altitude has been determined. However, this latter may be subjected to slight variations according to the distance and operational instructions adopted, with the result that the two graduations are not strictly proportional. The time origin is offset to O', to the left of O by an amount corresponding to the running time during which the aircraft is not heading towards its destination, *i.e.* manoeuvring time before take-off (segment $O'U$) and approach and taxying times after landing (segment UO)*.

The bend in the fan-shaped lines, related to the speed variation during cruising, always remains slight. The fan-shaped lines, which are practically straight lines going through the figurative take-off point U, allow us to introduce the wind: they are cut by inclined straight lines which are equal flight times.

The vertical scale at the left gives the fuel weights. Under the origin, the section OD is the amount of fuel used on the ground before take-off and not included in the maximum all-up weight fixed for the aircraft in flight. The segment OA is the remaining amount of fuel kept on board to allow for the descent from 5,000 feet during the final approach and for taxying after landing. Segment OM is the fuel used during the climb to cruising altitude and segment MD corresponds to descent from the cruising altitude to 5,000 feet, where the approach procedure begins. The continuous transverse line (K), which protracts the broken line, is the amount of fuel consumed during the cruising flight.

Now, what about fuel reserves? Firstly, we use, in France, a minimum margin of 10 per cent of the cruising fuel consumption. I shall be brief about the reasons for this reserve. It is intended to cover, on the one hand, maladjustment or wear of the equipment; and, on the other hand, errors of navigation and wind estimate.

*The total of these manoeuvres is taken here as 20 minutes, the minimum requirement. The combined time of 30 minutes, mentioned under Section 2.2 takes account of an additional 10 minutes for average stand-off time, the corresponding fuel being taken in the reserves.

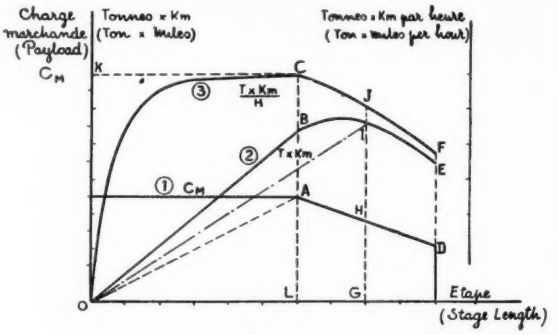

FIGURE 9.

This is represented by line (L) of Fig. 4. For stand-off and diversion, choice remains between two different methods:—

Either $\frac{1}{2}$-hour stand-off ($\frac{3}{4}$-hour for long range airlines) plus a reserve margin actually computed in relation with true distance of the alternate airport; the scale in the top right hand corner of the diagram allows a quick graphical computation (line (M) of the diagram).

Or an outright margin of 1·30 hour at the minimum hourly consumption; this condition is given by the line (N).

The maximum quantity of fuel usable in the aircraft is given by a horizontal limit which may be subdivided, if appropriate, into limit with normal fuel tanks (line O) and limit with extra fuel tanks.

The vertical scale on the right side, whose graduations increase in a direction opposite to that of the left-hand scale, gives the available payload.

The broken dotted-line gives, by way of example, the corresponding computation for each aircraft for a stage length of 1,900 miles, assuming it is flown with a 50 m.p.h. headwind.

For the Armagnac (Fig. 7) one may read off a flight time of 9.20 hours. The weight of fuel normally consumed is 13·1 tons. With the 10 per cent. reserve and the stand-off margin fuel load it increases to 15 tons; assuming a diversion distance of 310 miles, the fuel load rises to 16·5 tons, leaving a payload of 10 tons.

In the case of the Constellation (Fig. 8) we have given an example of the effects consequential to the sequence adopted for the cruising speed; curve (K)

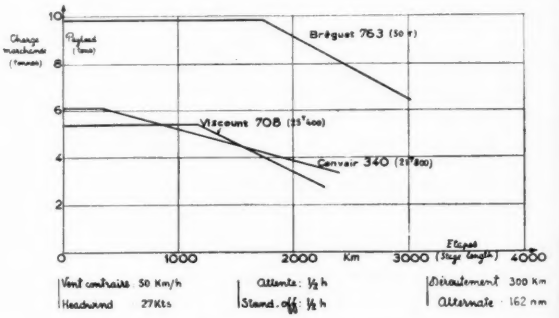

FIGURE 10. Medium range aircraft.

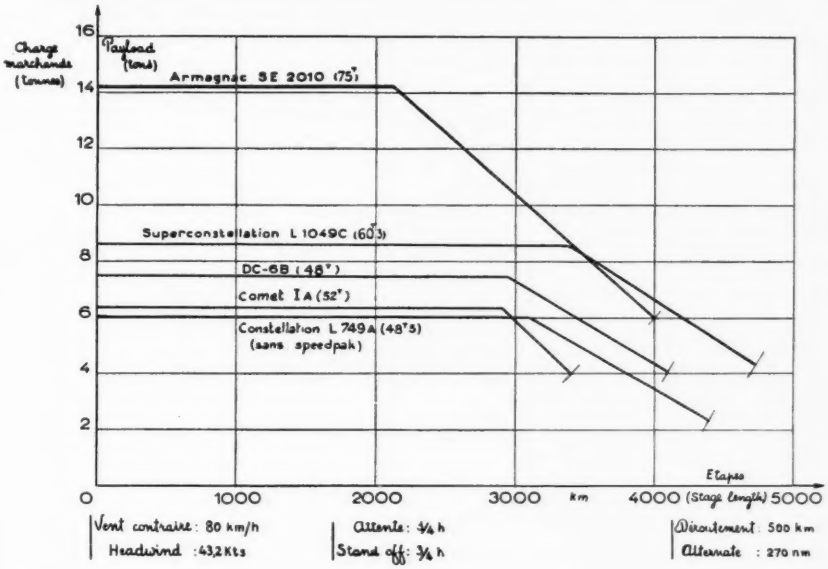

FIGURE 11. Long range aircraft (first class).

corresponds to the fastest sequence generally used at 20,000 ft., dotted line (K') to slowest sequence used at an altitude of 10,000 ft. (for which a special time scale has been drawn above the abscissae).

All-up weight limitations are evidenced in various ways:

Maximum A.U.W. at take-off appears in the offset which has been shown between the left and right vertical scales when drawing the diagram.

Limitation due to the zero fuel weight appears readily in the lowest portion of the transverse lines of the diversion graph (line (Q) of the diagram).

The same holds true for the capacity limitation, when it is lower than the preceding (line (R) of the diagram).

Maximum landing weight must be checked for each stage.

Study of the preceding diagram allows the plotting of a curve showing maximum payload (C_M) carried for any given stage (Curve 1 of Fig. 9) as a function of stage length and, of course, with the wind assumption made. The horizontal part PA is related to limitation due to zero fuel weight (or to aircraft volumetric capacity, if more restrictive). The leg AD reflects the normal decrease of payload when the fuel increases to cover the considered stage (limitation due to total take-off weight). For piston-engines, the curve is fairly straight; its curvature depending on that of line (K) in Fig. 4; it becomes rather noticeable with gas-turbines. Point D gives the maximum commercial endurance of the aircraft, all fuel reserve requirements being satisfied.

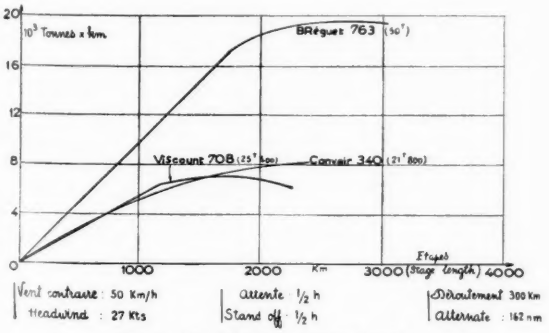

FIGURE 12. Medium range aircraft.

From this, Curve 2 can be plotted, giving aircraft ton-miles capacity in terms of stage length with maximum payload. Except on leg OB, this curve is not the integral of the preceding one. For any stage say, OG, the ordinate GI is the product of GH by OG; ton-miles on this stage will develop along the curve OI and not along OBI.

Lastly, it is a simple matter, by dividing by flying time, to plot Curve 3 giving ton-miles per flying-hour. For any stage shorter than OL this curve might be merged with line KC, should the plane be catapulted to cruising altitude and speed right away. Time losses arising from take-off, landing, climb to, and descent from altitude, affect commercial speed and hence en route time, proportionately more if stage-length is shortened. This results in strongly incurving Curve 3 which must now pass through the origin. This line shows also, for a given aircraft, variations versus stage length of a function which might be called hourly profit earning capacity; that is to say the ratio of revenue to direct hourly operating costs alone (*i.e.* with the exception of fixed costs, and landing and take-off costs), since

$$\frac{\text{Ton-miles}}{\text{Flying time}} \times \frac{\text{Mile-fare}}{\text{Hourly cost}} = \frac{\text{Revenue}}{\text{Hourly operating costs}}.$$

It must be pointed out, however, that the ratio of mile-fare to hourly cost is constant *for a given aeroplane only*. The numerator of this ratio is, of course, common

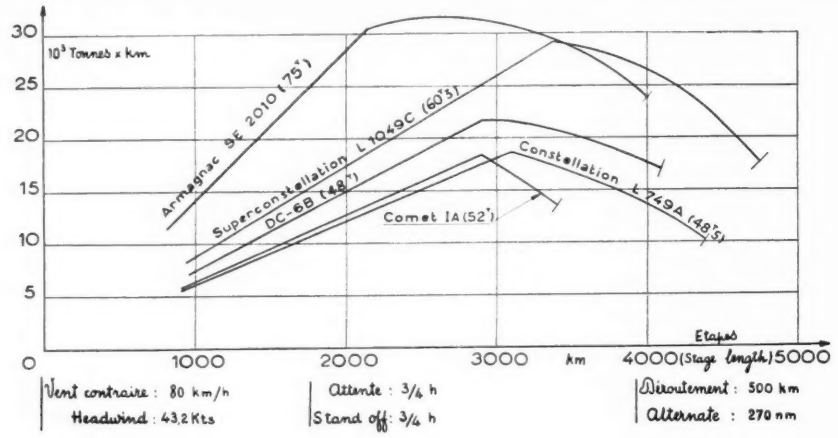

FIGURE 13. Long range aircraft (first class).

to all aircraft in a given commercial class, but the denominator depends on the aircraft involved. Thus, Curve 3 enables us to determine the optimum stage length for the specific craft in question. The curve, in and of itself (*i.e.* intrinsically), however, yields no comparison between non-identical aeroplanes, except to show what relation must be established over a given stage length between their unit hourly cost to equalise their hourly profit-making capacities for the stage under consideration.

As an example, Figs. 10 to 15 give respectively the Curves 1, 2 and 3 corresponding to some existing commercial aeroplanes, arranged under two headings, depending on their range, as follows:—

(*i*) Medium-range (Convair 340, Viscount 708, Bréguet 763).

(*ii*) Long-range (Constellations 749A and 1049C D.C.6B, Comet 1A, S.E. 2010 Armagnac).

Within each of these two headings, an effort has been made to formulate assumptions as uniform as possible for headwinds, stand-off time and diversion length. However, in the case of long-range aircraft, the average stacking time was reduced from 45 minutes to half an hour for the Comet to take into account its highest fuel consumption and the landing priorities which might result therefrom.

With this comparison the predominantly technical approach to the problem is terminated: going farther would need the introduction of commercial and competitive data.

To obtain curves which can be useful in comparisons of the so-called hourly profit earning capacity, for different aircraft, we should have to divide the ordinates of Curves 3 by hourly costs, which vary with each airline. The effect would be to regroup these curves. They are, in point of fact, scattered according to gross weight, the hourly costs being obviously higher for large aeroplanes than for small ones.

Furthermore, to obtain the *overall* profit earning capacity for the craft under consideration, we should have to add take-off and landing costs, and fixed costs to expenditure. Fixed costs can, if necessary, be introduced into hourly costs on an assumption about the yearly utilisation of material, but such an assumption is *a priori* necessarily artificial.

2.4. EFFECTS OF STOPS

With the help of the curves already given, one may determine for any given aircraft the distance beyond which it becomes economically sound to provide a technical stop for refuelling. A plot of stage distance versus aircraft ton-miles (Fig. 16, Curve 2) shows that the latter are, as we have seen, equivalent to revenue, except for the parameter of fares. Thus, on the left ordinate axis, which is purely financial in nature, we can superimpose in a negative form the additional cost due to a stop, F_E. If we draw from point L a line parallel to line OB, the resulting line LC represents what remains in revenue after one stop has been made during the trip (by deducting the cost of the stop, all other expenses being unchanged). The point A at which this line intersects Curve 2 is the point of equal revenue, whether a stop has been made or not, and gives the maximum stage distance which is desired for economic operation. Of course, for any stage distance higher than OH, such as OJ for example, the technical stop will be made practically no later than abscissa OG of point B, so as to get the benefit of maintaining payload at its maximum value. The ton-miles produced will continue to increase along the extension of line OB, but those represented by

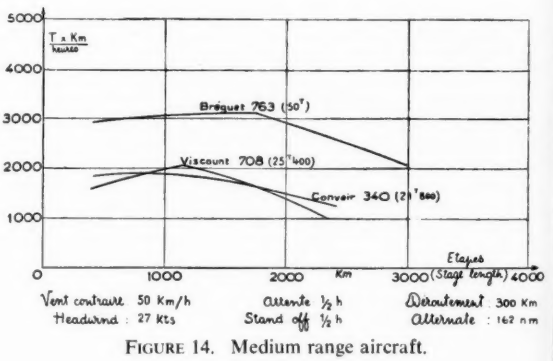

FIGURE 14. Medium range aircraft.

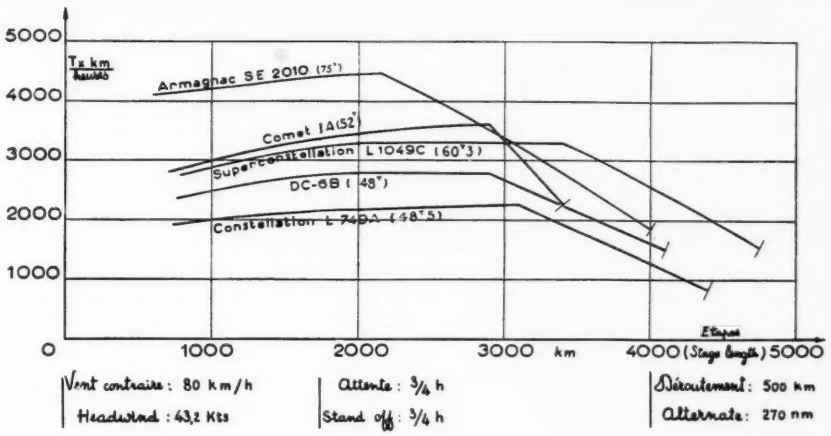

FIGURE 15. Long range aircraft (first class).

line CA will be called "pay-ton-miles." These are shifted from the preceding ones by a quantity representing the amount of ton-miles whose revenue has been offset by stop costs. Curve 3, giving the hourly profit-making ability shows a discontinuity DE at the stop point due to stop costs, and continues to rise asymptotically, relative to the extension of its portion OD, through point K of the same abscissa as point A on the initial Curve 3.

In Fig. 17, this graphical method has been extended to the commercial stop situation. Two such stops have been assumed on the distance OH, which as already shown was the maximum stage distance for economical efficiency. The "pay-ton-miles" are again represented by the line OK; they experience two interruptions at stops I and J. Of course, advantage will be taken of the second stop to make a partial refuelling. By this method we maintain the full payload HM, instead of the reduced load HN and lose nothing more than the revenue PA or $(-F_E)$, and not the revenue PK or $(-2F_E)$ which latter would be the case were the total flight distance equal to, or less than, OG.

In the same manner as the preceding analysis, this is to be understood only under the assumption that a load factor of one is achieved. As this will generally not be the case in practice, commercial stages can considerably influence the payload actually transported (Curve 1 in Fig. 18). Incorporating a commercial stop will generally improve the load factor on at least one portion of the flight distance; to make it pay, however, the additional ton-miles so produced should cover the staging costs*; furthermore, prestige considerations might induce operators to maintain it even with negative results. In any case, no general rule, even of a statistical nature, can be given *a priori* for an estimation method applicable to general case. The matter is essentially one of commercial traffic analysis bearing on each route.

*Including here the commercial costs due to passenger and freight handling. As they are related to commercial traffic, they were purposely discarded in Section 1.1 and did not fit in the preceding section concerning technical stages.

3. Utilisation

3.1. INTEREST IN THE PROBLEM

The fundamental criterion for annual utilisation of a given aircraft is defined as the time flown in hours during one year. Daily utilisation is also often considered, *i.e.* the number of hours flown in a 24-hour period, but this figure is only of interest and value when it represents an average, *i.e.* if it is the quotient of the total number of hours flown in one year divided by 365 days. Effective daily utilisation, *i.e.* the number of hours actually flown each day, although subject to wide variations, should be for the most part well above the average value because of repair and overhaul standing periods.

Increasing utilisation is an objective of prime interest if aircraft are to be made to pay dividends; for, all other factors remaining unchanged (and especially speed), it permits at the same time

(*a*) a proportionate increase in revenue;
(*b*) a proportionate decrease in the depreciation charge.

Economically speaking, the result is a "no cost" increase to the total fleet; a gain of 10 per cent. utilisation when operating a given type of aircraft is equivalent to presenting the company with a gift of one aircraft for every 10 aircraft of that type in its fleet.

Taking into account the fact that the depreciation charge amounts to nearly 1/7 of all operating costs when assuming a mean value of hours flown per annum, a 10 per cent. increase in utilisation (all other factors remaining equal), results in an increase of between 11 and 12 per cent on the ratio revenue/expenditure.

What are the main factors affecting aircraft utilisation ? Let us consider, on the one hand, those which affect grounding times; and on the other hand, those which govern flying times.

3.2. GROUND TIME

3.21. *Maintenance*

Maintenance is the most frequent cause of aircraft grounding. Improvements in this aspect can be obtained in two different ways; one bearing on the design itself, the other on workshop methods and organisation.

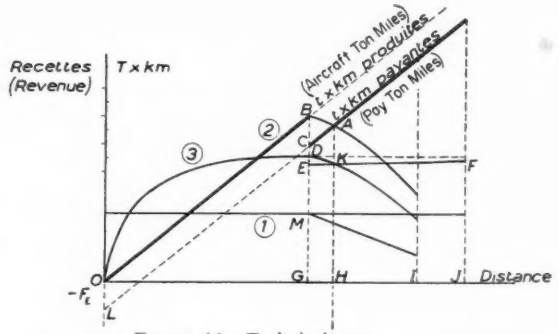

FIGURE 16. Technical stop.

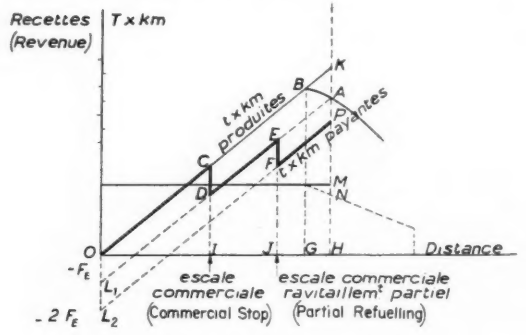

FIGURE 17. Commercial stop.

The former is particularly significant with regard to our general standpoint. Designing staffs must pay great attention to quick and easy accessibility from a maintenance standpoint. All available ingenuity can and must be applied in this field; much has already been achieved, but a great deal remains to be done.

The elegant solution called " part-exchange " is noteworthy in this regard. It consists of replacing a complex assembly (as one unit) by another one previously revised and overhauled in the shops; in this manner the aircraft as a unit will continue to fly and hence will bring in revenue and counterbalance the " writing off " of its corresponding investment.

As engineering progresses and becomes more and more elaborate, the list of such assemblies extends every day. The following are only a few examples:

Complete power-plant;

Main or auxiliary undercarriages;

Instrument panel;

Cabin air-conditioning assembly;

Automatic pilot;

And, of course, all radio equipment, which has always been considered quickly removable.

All this is easy to say, but its putting into practice is considerably slower. Like genius, as described by Edison, it consists of 99 per cent. perspiration and one per cent. inspiration. Incidentally, this one per cent. is a blood brother to Christopher Columbus's egg; it seldom wins for its author any acknowledgment from the uninitiated, to whom everything that is simple always appears as commonplace—once it is achieved.

Improvements to be obtained from a better organised maintenance shop are somewhat beyond the scope of this paper. But this problem also benefits from the developments of part-exchanges and interchangeability concerning the aircraft as a whole. An even more subdivided analysis, extending to as many subassemblies as possible, is requisite for highly rationalised subdivided shop work.

3.22. *Stops*

Utilisation depends considerably upon the extent of the lost times, which are represented by stops. These dead times can be shortened by a careful study of servicing times, and here servicing should be understood to include not only refuelling (which has been an active concern of development departments for some time) but also passenger servicing which, although formerly amply covered by refuelling time, may now often exceed it if not subject to adequate supervision. The problem is insidious, for refuelling procedures are gradually improving while aircraft passenger capacity is increasing.

Quick accessibility for passengers and luggage plays an important role and demands not only specific consideration at the design stage, but also the much-discussed investments for aerodrome handling equipment.

There remains the capital question of the number of stops.

For long-range aircraft, where stops are often a simple matter of refuelling, the frequency of stops turns mainly upon the range performance of the aircraft. In this case their number can be considerably reduced by employing artificial means, such as the flight refuelling procedure which will be mentioned later. It should be kept in mind, however, that the number of stops may also depend upon other factors, such as an exchange of crews to avoid excessive fatigue resulting from prolonged continuous duty.

For medium-range aircraft, the number of stops depends mainly upon economical geography, *i.e.* the area pattern to be connected. If this consists of short routes, as is often the case in Europe, the situation will be the reverse of that for long-range aircraft, and utilisation considerations would lead to an increase of the number of stages, or in other words, the number of trips in order to fly the aircraft as often as possible.

In fact, when the traffic volume available for a nearby destination is light and justifies, say, only one two-way trip each day, it would be convenient if the aircraft could be operated on some local traffic in the mean time instead of standing idle on the ground until its scheduled time for the return flight. This solution is unfortunately impaired, in international service, by the stringent rules which govern the traffic rights, namely for cabotage; but it has many applications in national networks (" W " pattern).

When the traffic volume is heavy, the situation calls for a shuttle service. Utilisation then depends upon

optimum use of each hour of the day, with emphasis on the slack hours; hence the reduced fares which companies have a tendency to offer as an incentive during off-peak periods. Goods or mail transport can complete this type of compensatory action and account for profitable returns, assuming adequate aircraft versatility, which will be dealt with thoroughly later.

3.3. FLYING TIME

Turning now to a study of aircraft flying time, we are faced with a peculiar aspect of the speed problem. At first sight, an increase in speed seems to produce a favourable effect on dividends, because it increases productivity for a given utilisation; in other words, if utilisation remains constant, revenue per mile increases with speed.

The problem, however, is to maintain this condition, *i.e.* to maintain constant utilisation. The situation is alarming even on the score of basic principles: geographical distances being obviously immutable, if speed is increased, flying times will necessarily decrease. Assuming that grounding times remain unchanged we are left with a proportional and inexorable decrease of utilisation. How exacting technical progress can be ! A gain in speed will pay dividends only under the absolute condition that it is accompanied by a corresponding reduction of the dead times or grounding period already analysed. This ideal is unfortunately far from having been attained.

A further complication arises because the travelling public is reluctant to accept arrivals and departures outside convenient hours. Thus the time element, although technically homogeneous becomes psychologically heterogeneous. For practical purposes, any departure or arrival between midnight and 8 a.m. must be abandoned. The situation is made more complex in east-west trips because of the disparities in local times at both ends of the route. Add to this the frequent dis-symmetry of prevailing winds and we have a vague notion of the magnitude of the task of preparing schedules for an air transport company wishing to maintain a reasonably intense utilisation. This problem has been dealt with by Mr. Christopher Dykes[2].

In general we cannot tell *a priori* what consequences will follow an increase of speed in this problem. For example, if in a given service a slow aircraft is operated on a two-way daily trip, the use of a high speed aircraft might sometimes transform this service into a more profitable "W" pattern—assuming additional traffic is available in the vicinity. If no such traffic exists, however, operating a high-speed aircraft on the same simple two-way daily trip will cause a decrease of the flying hours and hence of utilisation. Here again, the situation must be thoroughly analysed in each particular case.

In conclusion, let us merely point out that, if speed appears at first sight as a friend of dividends, it is in fact a treacherous friend. Those engineers who surpass themselves in battling weight and drag for sheer love of speed, their chosen Muse, might do better to meditate upon the words of disillusioned Talleyrand:—

"Seigneur, délivrez-moi de mes amis, mes ennemis, je me'en charge."*

4. Versatility

We have seen in Section 2 that an aircraft must be very carefully adapted to the route on which it is operated. An analysis of utilisation showed us that, on the other hand, it may be advisable to operate an aircraft on several routes in order to collect additional utilisation from different sources. Let us examine thoroughly this apparent conflict to see how nearly we can approximate the old dream of the general purpose aircraft.

4.1. INTEREST AND DIFFICULTY OF THE PROBLEM

Nobody will deny that airline operators have the greatest interest in standardising their fleet as far as is possible; a study of the accounts of various airline companies speaks volumes on the subject, and, other things being equal, it shows that an unhappy mixture of several aircraft types induces a heavy burden. Even without mentioning the advantages of greater flexibility for assignment and training of air crews and in simplification of the work of the airline operational personnel, standardisation of the fleet affords a significant economy as regards maintenance; and, correspondingly it permits the reduction of spare parts and the improvement of utilisation times. In this respect the trend of the United States airlines is typical: American Airlines which uses only two aircraft types, the DC-6 and the Convair, both of which are powered by the same engine type, the 2800 Pratt & Whitney, has for some time been cited as a leader in the field. Other airline companies, United Air Lines in the United States, and S.A.S., Sabena, and so on, in Europe are following step.

To achieve this goal, sizable difficulties must be overcome; and they vary in acuteness for each company. It is very difficult to restrict oneself to two aircraft types only, when the range of stage lengths over which the aircraft operate stretches from the few hundred miles separating the European capitals to the 1,900-2,500 miles requisite to trans-oceanic flights and even to 3,800 miles, in the case of certain major "potentially non-stop" flight routes.

On the other hand, owing to the seasonal character of the traffic, it is desirable to switch aircraft from one route to another (assuming that different routes have different peak periods) in order to use the aeroplanes at maximum load capacity. In addition to seasonal fluctuations, significant variations in load factor render a flexible adjustment in accordance with carrying capacity theoretically desirable in order to meet best traffic demands. Large and irregular meteorological variations of the type encountered over the North Atlantic require important fluctuations of the fuel load on the same principle.

*Lord, deliver me from my friends, my enemies I can take care of myself.

To satisfy these diverse requirements, the theoretical ideal is not, of course, to keep in readiness in hangars a complete range of various aircraft (*i.e.* in diametric opposition to the principles of utilisation) but rather to find an aircraft with flexible capacity, all-up weight, wing area and power: this is a fancy that even Jules Verne and H. G. Wells did not dare to dwell upon.

4.2. SPECIAL DEVICES

Nevertheless, within the limits of the conventional aircraft formulae which we know today, steps can be taken in this direction.

4.21. *Wing-tip tanks*

Firstly one may suggest the introduction of easily removable equipment for carrying extra fuel, so that the weight of the corresponding tanks does not permanently burden the aircraft load capacity. The wing tip tank technique has been successfully developed on fighter aircraft; why should not we adapt this method for transport aircraft ?

From the point of view of aerodynamics, wing tip tanks are conveniently located and do not seriously increase the total drag of the aircraft. Although aircraft form drag increases, a partial compensation is afforded by an increase in the apparent aspect ratio and the resulting decrease in the induced drag. The final result is that with a careful aerodynamic design, it can be supposed that the lift/drag ratio governing the fuel consumption per mile would not be very much altered at the angles of incidence generally used in cruising.

The following table gives some details for turbo-jet military aircraft.

Aircraft	*Aspect ratio (geometric)*	*Cruise* C_L	C_L/C_D *without tank*	C_L/C_D *with tank*	*Ratio of tank diameter to wing tip chord*
A	4·9	0·30	8·6	8·6	0·43
B	4·6	0·35	12·2	11·3	0·40
C	6·4	0·30	12	12	—

From a structural strength point of view, the hanging of tanks requires some local reinforcement; but, on the whole, the weight so added at the wing tip will tend to alleviate the flying load and associated stresses. On the other hand, the stress is more severe when landing with full tanks, and a quick fuel release or tank-dropping system should be provided for emergency landings shortly after take-off.

With regard to aero-elastic effects, the addition of a substantial concentrated mass will, of course, change the approach to the problem: bringing the tank in a forward position relative to the tip section is an advantage inasmuch as the wing elastic axis has a tendency to advance forward as well; it is a disadvantage inasmuch as there is a decrease in the frequency of the torsion-predominant fundamental modes of oscillation. Conceivably there exists an optimum position determinable for each particular aeroplane.

4.22. *Additional jet engines*

For those aircraft—and they are many—whose carrying possibilities are directly limited by the I.C.A.O. safety requirements for take-off conditions, installation of additional jet engines can be considered as well: mounting of engines and requisite tanks should be performed in less than half an hour, should the need so arise.

This system is particularly appropriate to twin-engined aircraft. Their all-up weight is severely limited because of the minimum angle of climb required after passing over the 50 ft. obstacle. They are thus unable to take full advantage of modern runways of 5,500 to 6,500 ft. With the additional jet engines this angle of climb requirement can be fulfilled with such a wide margin that the weight is then, in general, limited only by the length of the runway.

This system is especially advantageous for improving the potentialities of existing aircraft. The whole of the additional power can be used, in case of failure of one engine, in support of the remaining engine, the net effect being equivalent to a power increase double that of conventionally installed additional power. For example, a 2 × 2,000 h.p. twin-engined aircraft with an additional 400 h.p. equivalent power jet engine has available, in cases of main engine failure, a latent power of 2,400 h.p. This equivalent could only otherwise be achieved by mounting two 2,400 h.p. engines, thus raising the installed power by 800 h.p., with all the attendant weight and costs consequences. The well-known lightness of jet engines makes them an ideal device of power reserve to be installed as safety precautions against the undesired, but all too frequent, emergency situation.

Very full tests on the French twin-engined aircraft SO.30P show that mounting two Turboméca "Palas" jet engines giving 350 lb. thrust each gives to this aircraft a gain of 3,100 lb. at take-off, whatever the season may be. The weight of the installation is less than 650 lb. Allowing 220 lb. for extra fuel consumption we have a payload gain amounting to 2,200 lb. This represents 11 passengers with luggage or an increase in full load range of about 560 statute miles.

The installation of such additional jet engines could be equally pertinent for those types of contemporary jet aircraft which frequently experience difficulties at take-off, especially during hot weather. However, the interest in such installation is mainly as regards improving existing aircraft. For new projects, it might be more advantageous for fuel consumption economy to increase thrust in cruising and hence flight altitude by installing at the outset more powerful main engines than strictly required. This, of course, would have the subsidiary effect of improving take-off capacity without the use of additional secondary jet engines.

4.23. *Flight refuelling*

The most important revolution in the field may come from refuelling in flight. With the remarkable trans-Atlantic tests conducted by Flight Refuelling Ltd. and B.O.A.C. during 1948, the round-the-world non-stop flight made by the Boeing B.50 "Lucky Lady II"

during 1949, let alone the operational tests conducted more recently in the United States (particularly during the Korean War) this technique seems now to be well-enough proven from the point of view of safety to permit its commercial use being contemplated in the near future. On purely technical grounds, this process is very promising with regard to maximum flexibility in tailoring the range to the stage length; it could theoretically enable non-stop flights to anywhere with medium-range aircraft, taking the precaution of selecting the four-engined variety for better safety on trans-oceanic or desert routes. One would then easily be led to indulge in imagining a completely standardised fleet in an airline using a single type of aircraft of that kind.

Unfortunately, the tantalising dream of a complete homogeneous fleet, so tempting to every airline maintenance chief, is unattainable with contemporary medium-range aircraft. They do not allow sufficient range for oceanic crossings, taking into account the obvious necessity of reaching by themselves an emergency landing point, in case of failure of refuelling in flight. Even on predominantly land routes, such as Paris-Saigon or Great Britain-Australia, they are not roomy enough to permit long-duration flight free from undue fatigue for passengers or, more important, for crews. In truth, there is no necessity for non-stop through-traffic, and refuelling in flight is just the right way to divide the distance, as required, into long stages just fit for the full-time employment of one crew, with a shifting at each stop. As generally occurs, the best advantage of this new technique can only be achieved by aircraft specially designed for the purpose.

It is in the field of profit-earning capacity that this solution raises the nicest problems. The refueller's flight is costly. Where stops are not necessary from a commercial point of view (*i.e.* stops are made solely because of refuelling needs), but under present conditions frequencies are low, the annual utilisation of the refueller would be rather small. It is worthwhile, however, to study closely the balance to be arrived at from a consideration of the costs as against the savings inherent in a refuelled aircraft from the cancellation of landing and take-off charges, the improvement of overall utilisation, the maintenance simplification derived from the resulting consolidation of the fleet, and most of all the gain in payload won from the enormous and ever-increasing waste transportation of fuel load by modern air liners.

4.3. GENERAL LAYOUT OF AIRCRAFT

Even without innovation and contenting ourselves with conventional extant aircraft, the quest for versatility leads to useful thoughts and effective results, however obscure and unobtrusive at first appearance.

The first consideration in this respect is the mission to be assigned to the aircraft: shall it be intended for passenger-and-luggage transportation only, for freight transportation only; or is a composite solution to be contemplated and to what extent?

Every airline operator knows what a key role is played in passenger-aircraft profit-making capacity by additional freight and, particularly, by first-class freight, *i.e.* mail. On the other hand, the operators also know how difficult it is to adjust the flow of freight in order to arrive at optimum profit-making capacity. They cannot rely only upon demand, for this variable may all too often operate to increase the very item, *i.e.* passenger fluctuation, which such freight is intended to minimise. This problem is that of delayed freight, and it does not fall within our framework since it is above all of a commercial nature: suffice to say that, fortunately the problem is simpler to solve for long routes, where its importance is greater, than for short and medium routes. In fact, a compromise is necessary for all passenger aircraft, and especially for long-range aircraft: it is left to the ingenuity of both manufacturer and operator to season this mixture with moderation and tact, so as not to mar attractiveness nor to give the comfort-seeking passenger the unpleasant suspicion that he is travelling in a cargo aircraft.

This problem should be considered by the aircraft manufacturer when calculating cargo compartment volume. In some instances, *e.g.* the Constellation "Speedpak," it appears that this factor was not given due emphasis during the inceptive stages of design. The required minimum is that the mean seat occupation percentage allowed for passengers in the computation of provided capacity (65 per cent. for instance) be 100 per cent. utilised. But the ideal situation would be that in which the entire available aircraft payload capacity would be usable for freight over a stage length corresponding to maximum operating economy. But here again we are limited by the cost involved: each extra cubic foot increases the drag and structure weight. A recent analysis made by the Société Bréguet indicated, by detailed comparison between two designs indentical except for volume, that for a 38 ton three-engined jet aircraft whose cruising air speed is 500 m.p.h., a 28 per cent. increase of fuselage volume permitting a 20 per cent. increase in the number of passengers, results firstly, in a 2·5 per cent. increase of the aircraft empty weight; and secondly, in an extra fuel weight ranging from 2 per cent for a 600 mile stage-distance to 3 per cent. for an 1,800 mile stage-distance. On distances of less than 600 miles the payload is increased by 2·5 tons (or 6·5 per cent. of all-up weight) by raising the capacity limit, and on distances above 1,200 miles the payload is reduced by 2,000 lb. (or 2·4 per cent. of all-up weight) due to the larger structural weight and fuel load.

Another important feature is the provision of convenient access to luggage compartments, thereby allowing rapid and discreet loading and unloading at staging points without disturbing the time-table: passenger transport aircraft are almost always in a hurry.

Now, if we start from the other extreme, *i.e.* the cargo aircraft, we will find it very difficult to resist taking the advantage of its flight (and this is especially true in economically under-developed countries) by carrying passengers who are, under present tariffs, the most profitable load. On a mile basis the average passenger fare is twice that of the equivalent weight in freight and half the price of the same space occupied by freight.

Certain types of bulky, cumbersome or unpleasant freight is unsuited to this mixing. Moreover, in economically developed countries, the passenger being more pampered is all the harder to please and will not accept a secondary status. With freight transport open to a semi-industrial kind of development, a preference for independent cargo and passenger operations may be in the offing.

It is suggested, by way of conclusion, that until a significant increase in air freight is foreseen, separate passenger transport cannot be considered. But the converse is not true. Moreover, in regions where surface transport facilities are deficient, the true passenger-cargo aircraft still has bright prospects.

5. Amortisation

Annual amortisation (depreciation) of aeroplanes, which accounts for 12 to 14 per cent. of direct operating costs—within the meaning the term has been assigned—is the quotient of prime cost less residual value (or resale price) by the aircraft life. Let us look at these items.

5.1. PRIME COST

Prime cost depends strongly on the number of aircraft produced in a batch, and this for several reasons.

First of all, design, manufacture, test and development costs of the prototype are to be parcelled out to all aircraft of that design: the more they are, the less the proportional share. These costs are roughly proportional to the take-off gross weight of the aircraft. For an equivalent gross weight, the higher the performance, and the better the detailed aircraft design (especially regarding maintenance and operation convenience, the importance of which we have already pointed out, as far as turn-round and versatility are concerned), the higher the cost.

Nowadays, and until air transport is fully self-supporting financially, this charge is often borne by governments, in order to foster the development of national technique. The United States themselves, the most outspoken supporters of integral liberalism in this field have been, not so long ago, progressively subsidising, at least indirectly, civil aircraft prototypes which formerly were traditionally financed only by airline operators themselves. Let us then say no more about this cost item. Its components are somewhat imprecise, since it is difficult to define the exact scope of the field; take power-plant development, for example.

Production aircraft costs are decreasing materially as the number of aircraft ordered increases. This is the principle of the "learning curve." One may say that the unit cost of an aircraft is lowered by 18 per cent. every time the number of aircraft in a series is doubled. The foregoing implies, of course, the basic condition that continuous production is in question, with no delays in supply and an efficiently functioning assembly line.

In short, the unit prime cost is given by the following formula, as against production aircraft number N,

$$\frac{p_0}{N} + \frac{p_1}{N^{0.25}}$$

where p_0 represents the prototype cost and p_1 the cost of first production aircraft.

A basic element of profit earning capacity is therefore to obtain as large a mass production as possibile. The only really normal and sound means to obtain this is to design an aircraft versatile enough to be selected by a good many airlines, even assuming different specific needs: the importance and implications of this question have been dealt with already.

A good help, somewhat artificial, but nevertheless effective, is the support to be afforded by important military orders, with due consideration being given to suitable harmonisation of requirements, this aim being most likely to be reached when the civilian type is versatile. Such mutual aid was at its peak during the war. This explains largely the exceptionally favourable place gained on the market by American aeroplanes, such as the Douglas DC-3, widely mass-produced for the military in the C.47 and C.53 versions, and the Douglas DC-4, of which the military version, the C.54, boomeranged to the military pocket-book, the sharp vertical rise of second-hand prices for the civilian counterpart at the outset of the Korean War. In France, the Nord 2500 freighter is to be manufactured under "siamesed" orders, and the case has been the same in Great Britain, as far as I know, for Vickers Valettas and Vikings.

When such a combined production is not feasible for the entire aircraft, it can be realised at least for major assemblies. The most ingenious example in this respect is that of the Boeing Stratocruiser whose wing and power plants were borrowed from the mass-produced B.50 bomber, while its fuselage came from the military cargo type C.97.

5.2. AIRCRAFT LIFE

In the past, aircraft had a limited life calculated in terms of thousands of flying hours. Overhaul and replacement techniques have now reached such a degree of efficiency that by undergoing specific periodical inspections (some of which are very expensive), an aircraft can be operated indefinitely and need be withdrawn from service only when it becomes obsolescent. Unfortunately, this still occurs quite quickly, especially in comparison with other types of vehicle.

This supplanting is most generally the effect of the international technical competition, which connection partially justifies, or at least explains, the policy of governmental subsidies.

The extension of aircraft life can be achieved in three non-mutually exclusive different ways.

The first is to start with a generous technical advantage. This method is attested by the negative alternative: all countries had experience with aircraft, which although undeniably in advance of their predecessors have nevertheless been relatively short-lived because the margin of progress was too narrow and they were consequently quickly outdistanced. A really great stride forward is the "royal flush" of aviation but this type of progress precludes the fear of backing revolutionary design: the Comet is one of the brightest examples of this policy.

As world champions do when they improve their own records, the aircraft designer must keep up his supremacy. This is the focal problem of aircraft modernisation. In view of this, definite possibilities must be expressly provided and reserved.

For example, on aircraft fitted with propellers, a provision for large propeller clearances will permit the installation of more powerful engines without changes in landing gear and wing geometry (DC-6 and Constellation, for example). Fortunately, this precautionary measure does not apply for jet-propelled aircraft. On the other hand, to allow a fruitful increase in range or payload, the all-up weight of a successful aircraft must have potential "stretch." Some ingenious means may be used to increase capacity, *e.g.* lengthening the fuselage to convert the Lockheed 749 into 1049 and DC-6 into DC-6B. Intensive utilisation of a long-developed formula induces relationships offering two advantages: a low production cost, and a progressive improvement in response to operational experience.

The third means of increasing longevity is to get the aeroplane written off the books. It is actual market value which is realisable by the airline (perhaps at a price of some lowering in class) after the craft has been completely amortised. With respect to the amortisation we are dealing with, this may be considered as a problem beyond the grave. Solving it, is solving by the same token the problem of the residual value, or the resale price.

Here again we meet the essential importance of versatility which enables an aircraft to be used for tasks different from those for which it was initially intended. In this field the two main considerations are available volume (for there is always a lack of volume) and take-off ease (even at a slightly reduced all-up weight) in order to allow operation from smaller airfields. In this respect, let us point out the soundness of the British policy of not overburdening wing loadings, which is in opposition to other technical policies, especially across the Atlantic.

From the point of view of profitable operation the question of change of class is one that sets many problems: one must acknowledge that a new aircraft, although quite expensive, if also economical for operation as a second line aircraft, may supersede a first line veteran, even completely amortised, if in direct operating costs it saves more than its own amortisation, *i.e.* if its operating costs are 12 per cent. to 14 per cent. lower.

One might be tempted to take advantage of this 12 per cent. to 14 per cent. margin to forecast the life expectation of a first line aircraft faced with a new rival. Unfortunately, this is not always the case because a slight difference may be sufficient to enable a more attractive competitor to steal its passengers. In other words, from the standpoint of the two terms of the ratio we are considering, it is the numerator, *i.e.* the revenue, which is by far the more vulnerable and which in point of fact, may be submitted to a decrease of more than 15 per cent. by lack of customers.

This leads to the problem of attractiveness. Here we leave the sphere of figures to enter the far more unstable and imponderable field of taste and vogue.

6. Attractiveness

Pretty girls are not alone in drawing on every available resource of charm and seductiveness to establish and maintain their fealties. On highly competitive air routes aircraft have to cope with a rivalry of the same nature. Perhaps this is why you English have the charming habit of referring to aeroplanes and ships as "she."

We must therefore, when tackling this problem, pay due tribute to the beauty and luxury of décor, such features being as necessary as body lacquer or upholstery colour in selling an automobile—and that is saying a good deal! Operators are fully aware of this and they give these questions, as well as courtesy, meals, and so on, all the care they deserve.

But our fixation on technology will soon make clear to us some parameters which do not pertain to aesthetics.

6.1. COMFORT—AMENITIES

Such is the case with cabin air-conditioning, more and more necessary with increase of cruising altitudes. This requirement would have to be satisfied, even with cruising altitudes of only 10,000 ft., for at least two reasons: (1) to make extended flights possible without undue fatigue and (2) to make possible the acceptance of any passenger, without regard to physical condition and without medical examination; the latter procedure being, in truth, inconsistent with the present-day trend to speed-up ground operations and to shorten formalities. Many advances are still to be realised in that field. Specifically, smoothness of conditioning control is still to be improved to avoid the "scotch shower" we knew in the early days of high altitude cruising.

Sound and vibration proofing also present highly technical problems. I shall not labour them, since they are well known. Advances have been made by way of dividends from improvements in associated fields, *e.g.* the advent of the gas turbine.

Ground visibility is an all too often neglected factor in pleasant flying. Passengers do not appreciate being carried like parcels. They wish to have a good view, weather permitting; and it is not only because they want to be the sole survivor in the tail after a crash that they covet rear seats in a low-wing aeroplane. We are all familiar with a British aeroplane of great merit whose publicity is partly based upon advantages thus conferred —in this case—by her high-wing configuration. The problem, be the wing low or high, will become more and more difficult to solve with congestion of central areas of the cabin as a consequence of increase in capacity. Its solution is to be reached by enabling the passenger to leave his seat more frequently during flight, at the same time taking the numbness out of his legs. The Stratocruiser's lounge is relevant in this respect.

6.2. SAFETY

Safety is a primary preoccupation, though often concealed, of the passenger. We do not intend here to devote a comprehensive analysis to this question, a matter for which a whole lecture would hardly be

enough. Suffice to note some of its effects on profit-making capacity.

Safety must over-ride, in every case of conflict, the considerations of daring and novelty which are so important to aircraft longevity. This consideration will possibly effect the greatest delay in the access of public transportation to supersonic fields. These two prospects are not necessarily inconsistent with each other. Technical development is often able to join them harmoniously There are numerous examples. The advent of gas turbines, using a fuel less inflammable than petrol, minimised fire risks. High-cruising altitudes permit flying above certain meteorological disturbances and their dangers—but introduce a new risk, in case of a window failure. Range lengthening reduces the number of take-offs and landings, and all of the dangers inherent thereto which, according to statistical tables, account for nearly 70 per cent. of all risks affecting air transport.

6.3. SPEED

Air speed carries with it an undeniable prestige. From a service efficiency point of view, however, it is the commercial speed which pays the dividends along with reductions in the number of stops. This follows what we have already said in our study of the rate of utilisation.

Stops are in themselves a source of physical fatigue for the passenger because of the attendant descent and climb—at least so long as the cabins are not actually pressurised to the true ground pressure. If stops are too frequent, they cause inconvenient disturbances, especially for night services—nullifying the effect of the sleeping berths arrangement, which should become the rule in the future for such services.

Hence the practical suggestion that high speed aircraft could be used in the future for low fare, long distance, mass transport hops with night departures and arrivals, while de luxe services would be continued for slower long-range aircraft equipped with sleeping berths.

6.4. FREIGHTER AIRCRAFT

In the foregoing, the factors considered have been those pertinent to passenger transport.

For pure cargo aircraft, no question of attractiveness in the aesthetic sense arises, since the customers by themselves know little about the aeroplanes, and the goods have no tastes to express. Everything is dealt with by technicians, designers and operators, *i.e.* by the initiated. Attractiveness being a function of economic aspect is here irrelevant in the direct sense.

We deem it worthwhile to include a survey of those aspects of attractiveness which are relevant and which will be particularly appreciated by the operator, determining his choice in many cases.

The first of these is the volume of available capacity and its distribution. Experience shows that the provision for volume is never excessive: 180 cubic feet per ton of payload is a minimum requirement; it should be more than doubled whenever possible. Another consideration of almost equal importance is the floor surface area. Maximum extension is desirable to permit the best distribution of divisible cargo. This leads to a preference for multiple deck layout. Unfortunately, there still exist large, heavy, cumbersome and indivisible loads for which a continuous block capacity is an imperative requirement. A strong element of attractiveness for operators would unquestionably be a mobile intermediary deck.

Another key feature is the rapid handling of cargo during stops. A fine solution to this problem is the "Flying Container" proposed not long ago by Miles with his four-engined aircraft M.68. This solution is not as yet widespread because cargo transport has still its great development to come in the future; there seems to be little doubt but that a solution of this kind would meet with large success in the future, provided that it could be achieved without excessive tare weight.

In the absence of this convenience, it is at least essential that a generous number of exits large enough to permit loading of cumbersome cargo be provided, and that the aircraft itself be equipped with appropriate hoisting devices. In this connection, from an estimate made in 1948 by a Curtiss-Wright executive, and quoted in *Interavia*, freight handling costs were increased by about $2\frac{1}{2}$ dollars per ton solely because the height of the loading platform of trucks had not been standardised with the height of the aircraft floor.

Particular attention should be given to the pressurisation problem. An increasing variety of goods is being transported by air, and of these, many cannot withstand the high altitude which is becoming more and more necessary for economic operation of the aircraft. It is difficult to resolve this condition, with the requirement for ease and quickness of handling, without posing some neat tightness problems. This criticism is especially pertinent to the "Flying Containers."

Finally, the possibility of flying at low altitudes with the doors open should be mentioned. This will certainly be investigated in connection with the use of parachute dropping procedures by the military. I have already emphasised the usefulness of this solution for lowering purchase costs. Specific civil uses of this method are evidenced in the case of an unusually long load. For example, a spare wing for a DC-4 has been transported in a Fairchild Packet with its tip protruding a considerable length beyond the rear of the fuselage through the intentionally opened doors.

7. Conclusion

Although not exhaustive, the foregoing survey provides us with an acute awareness of the intricacy of the profit-making capacity problem for commercial aircraft. Each of the topics poses a distinct aspect of the problem; and, taken together, they constitute at the very least a creditable excuse for the present lack of mathematically definable and really valid criteria.

The ton-mile system of reckoning is still the best approach to the problem, but only because of the lack of anything better. Certain highly relevant factors, such as

utilisation and amortisation, figure only indirectly in this criterion, through *a priori* assumptions. Other critical considerations, such as attractiveness and versatility, are completely by-passed. These latter, still considered as comparative imponderables, depend for their assessment on the particular operator's flair, instinct, judgment; therein lacking the stability demanded by manufacturers in an industry whose inexorable fate is to develop, at considerable expense, a long-born short-lived craft.

Despite the multitude of difficulties, however, the financial position of most airlines is improving yearly. It is "tilting at windmills" to argue that this solvency results from improved administration alone, or in the alternative, from a more commercially efficient handling of equipment. On the contrary, one is constrained to state (after becoming familiar with the intricacies of the problem) that this monetary profit arises from a combination of both factors: a fair reward for the joint endeavours of operator and manufacturer.

Unchallenged financial self-sufficiency for the entire field of commercial aviation appears at times as a dream just out of grasp and at other times as an unattainable distant goal. It is for us to reply to the challenge with realism and enterprise . . . to essay the brand of forceful courage which Louis Bleriot displayed when he scanned the awful expanse of Channel to the distant, inimical cliffs of Dover . . . and to discover, as Bleriot did, that the goal once achieved is a welcoming and rewarding one.

Acknowledgments

The author wishes to express his deep appreciation of the help he has received from so many persons that he feels unable to quote them all.

He would like at least to mention:—

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2. DYKES, J. C. (1952). Some Operating Problems of Future Transport Aircraft. *Journal of the Royal Aeronautical Society*, March 1952.

MR. B. S. SHENSTONE (*Fellow*), proposing a vote of thanks to the lecturer, said they should all be very grateful to General du Merle for bringing to them such a fine view of the very difficult job of making airlines work and aircraft pay, and for bringing a lot of the problems into focus. However, in one place the lecturer had given the impression that if one knew how to do a thing the rest was easy, but it was difficult to agree to that; in fact in his own experience it was more difficult than was indicated in the paper.

One of General du Merle's remarks was particularly apt to almost everybody who ran an airline, and that was his reference to the "long-born, short-lived aircraft." That description would be understood by them all, but it was to be hoped that such aircraft would not always be with them.

In the original version of the translation of the lecture the phrase had occurred "we are here to deal with the linking of great nations in an intimacy of liaison." This felicitous phrase should not be forgotten.

Bleriot had flown to England but the result was that his example had been followed rather in reverse, for now far more Englishmen flew to France than Frenchmen to England. This was a great pity and it was to be hoped that this Coronation year would see a reversal of that trend, and that more Frenchmen than ever before would fly to England. Possibly if they once got into the habit of it they would come every year, which would be a very happy thing.

Following the Lecture a Dinner was given by the Council of the Society at 4 Hamilton Place, at which the following were present:—

Mr. J. E. Adams, London Manager, Air France; Monsieur Jacques Allez, President, Aéro Club de France; Commandant L. J. Andlauer, D.F.C., Assistant Air Attaché, French Embassy; Mr. L. S. Armandias (*Companion*), Correspondent in Great

Britain, Association Française des Ingenieurs et Techniciens de l'Aéronautique.

Dr. A. M. Ballantyne, T.D., A.F.R.Ae.S., Secretary, Royal Aeronautical Society; Mr. E. C. Bowyer, C.B.E. (*Companion*), Director of the Society of British Aircraft Constructors; Lord Brabazon of Tara, M.C., Hon.F.R.Ae.S., Past President of the Royal Aeronautical Society, President, Royal Aero Club; Mr. John Brancker, A.R.Ae.S., British Overseas Airways Corporation; Monsieur J. Brocard, A.F.R.Ae.S., Directeur des Recherches des Ateliers d'Aviation Louis Breguet, President de la Commission d'Aérodynamique de l'Association Française des Ingénieurs et Techniciens de l'Aéronautique, 1949 Louis Bleriot Co-lecturer; Sir John S. Buchanan, C.B.E., F.R.Ae.S., Past President, Royal Aeronautical Society; Major G. P. Bulman, C.B.E., B.Sc., F.R.Ae.S., Director of Construction and Research Facilities, Ministry of Supply, Past President, Royal Aeronautical Society.

Mr. S. Camm, C.B.E., F.R.Ae.S., Director and Chief Designer, Hawker Aircraft Limited, Member of Council; Dr. R. W. S. Cheetham, M.B., Ch.B., M.A., Ph.D., A.F.R.Ae.S., Vice-President, Southern Africa Division of the Royal Aeronautical Society; Sir Harold Roxbee Cox, Ph.D., D.I.C., B.Sc., F.R.Ae.S., Chief Scientist, Ministry of Fuel and Power.

Ingénieur en Chef N. Daum, Direction Technique et Industriel Ministére de l'Air, Paris; Mr. G. H. Dowty, F.I.A.S., F.R.Ae.S., President of the Royal Aeronautical Society.

Mr. G. R. Edwards, C.B.E., B.Sc., F.R.Ae.S., Director, General Manager and Chief Engineer, Aircraft Division, Vickers-Armstrongs Ltd., Vice-President, Royal Aeronautical Society.

Sir William S. Farren, C.B., M.B.E., M.A., F.R.S., F.R.Ae.S., Technical Director of A. V. Roe & Co. Ltd., Vice-President, Royal Aeronautical Society; Sir A. H. Roy Fedden, M.B.E., D.Sc., Hon.F.I.A.S., F.R.Ae.S., Member of Council, Royal Aeronautical Society.

Colonel Grimal, French Representative for Air on the Standardisation Committee in London.

Mr. S. Scott Hall, C.B., M.Sc., D.I.C., F.R.Ae.S., Director General of Technical Development (Air), Ministry of Supply, Member of Council, Royal Aeronautical Society; Mr. N. J. Hancock, A.F.R.Ae.S., Ministry of Supply (A.I.D.), Vickers-Armstrongs Ltd., Weybridge, Member of Council, Royal Aeronautical Society; Mr. A. S. Hartshorn, B.Sc., F.R.Ae.S., Scientific Attaché, British Embassy in Paris.

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Deformations and Stresses in Symmetrically Loaded Circular Plates of Varying Thickness

by

D. C. BOSTON

1. Introduction

The problem of symmetrically loaded circular plates of constant thickness is covered adequately in several of the standard textbooks on the strength of materials. Formulae are derived enabling the deflections and stresses to be readily calculated in any portion of the plate. The problem is complicated by the introduction of a variable thickness and by a variation of material properties due to a temperature gradient down the radius of the plate; conditions such as are encountered in gas turbine wheels, bearing support diaphragms and flexible disc couplings. Methods exist whereby the plate can be solved if the thickness is a simple function of the plate radius, but in practice this is not always so, the profile often being complicated by flanges and spigots.

The similarity between the differential equations governing the deflection of a symmetrically loaded circular plate and the growth of a rotating disc suggested that some of the methods available for the solution of a rotating disc might be adapted for the circular plate. The method to be described treats the plate as a series of concentric uniform thickness annuli. The boundary conditions at the junction of adjacent sections are then equated so that a solution for the complete plate is obtained in terms of the conditions at the inner and outer edges. By adjusting these to suit the existing conditions at the edges of the plate the final solution can be obtained. A sum and difference notation is used and the results are obtained from charts in a manner similar to the stressing of rotating discs with the Donath Chart.

The three systems of loading, pressure loading, shear loading and applied couples, necessitate separate charts. For a plate under a combined system of loading the results must be obtained separately and superimposed. The charts are curves of the sum and difference of the radial and tangential bending moments for a constant thickness plate plotted against the plate radius. For a plate of varying thickness the profile is divided into a number of sections, the choice of sections being arbitrary, although in general the sections should be closer together where sharp changes of thickness or material properties occur. The sum and difference of the radial and tangential bending moments over the first constant thickness section will follow the sum and difference curves on the appropriate chart. At the junction of the first and second sections, an abrupt change in the values of the sum and difference will occur, the values following the curves on the chart again over the second section, the process being repeated until the outer edge of the plate is reached. The change in the sum and difference values at the section boundary will depend upon the change of thickness and material properties over the adjacent sections and formulae will be derived for this.

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Notation

- ω deflection of plate (in.)
- ϕ slope in radial direction (radians)
- Q shear force in lb./in. of periphery
- Δ Flexural rigidity of the plate $= Eh^3/12(1-\mu^2)$
- E Young's modulus (lb./in.^{2})
- h thickness of plate (in.)
- μ Poisson's ratio
- r radial dimension (in.)
- M_r radial bending moment (lb. in./in.)
- M_t tangential bending moment (lb. in./in.)
- δS change in value of S in passing from one section to adjacent section
- δD change in value of D in passing from one section to adjacent section
- a outer radius of plate (in.)
- b inner radius of plate (in.)
- $\rho = r/a$
- $\lambda = r/b$
- σ_r radial bending stress
- σ_t tangential bending stress
- $S = M_t + M_r$
- $D = M_t - M_r$
- P total load (lb.)
- p distributed load (lb./in.^{2})

2. Basic Equations

The basic equations from which the following work is derived are

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d\omega}{dr}\right)\right] = -\frac{Q}{\Delta} \quad . \quad . \quad (1)$$

$$M_r = \Delta\left(\frac{d\phi}{dr} + \frac{\mu}{r}\phi\right) = -\Delta\left(\frac{d^2\omega}{dr^2} + \frac{\mu}{r}\frac{d\omega}{dr}\right). \quad (2)$$

$$M_t = \Delta\left(\frac{\phi}{r} + \mu\frac{d\phi}{dr}\right) = -\Delta\left(\frac{1}{r}\frac{d\omega}{dr} + \mu\frac{d^2\omega}{dr^2}\right). \quad (3)$$

Combining equations (2) and (3),

$$M_t = \frac{\Delta\phi}{r} + \mu\left(M_r - \frac{\Delta\mu}{r}\phi\right).$$

Therefore $$\frac{r}{\Delta}(M_t - \mu M_r) = \phi - \mu^2\phi$$

or $$\phi = \frac{r(M_t - \mu M_r)}{(1-\mu^2)\Delta} \quad . \quad . \quad . \quad (4)$$

Introducing the Sum and Difference Notation,

$$M_t = \frac{S+D}{2}, \quad M_r = \frac{S-D}{2},$$

from equation (4),

$$\phi = \frac{r[S(1-\mu)+D(1+\mu)]}{2\Delta(1-\mu^2)} \quad . \quad . \quad (5)$$

At the junction of two sections at radius r the slopes can be equated, *i.e.*

$$\frac{r[S(1-\mu)+D(1+\mu)]}{2\Delta_1(1-\mu^2)} = \frac{r[(S+\delta S)(1-\mu)+(D+\delta D)(1+\mu)]}{2\Delta_2(1-\mu^2)} \quad (6)$$

Equating the radial bending moments at the junction,

$$\frac{S-D}{2} = \frac{(S+\delta S)-(D+\delta D)}{2},$$

from which $$\delta S = \delta D. \quad . \quad . \quad . \quad (7)$$

Substituting equation (7) in equation (6) and simplifying gives

$$\frac{\Delta_2}{\Delta_1} = 1 + \frac{2\delta S}{S(1-\mu)+D(1+\mu)} \quad . \quad . \quad (8)$$

If there is no temperature gradient down the radius of the plate, E remains constant and Δ is proportional to h^3. Hence equation (8) reduces to

$$\delta S = \delta D = \left[\left(\frac{h_2}{h_1}\right)^3 - 1\right]\left[\frac{S(1-\mu)+D(1+\mu)}{2}\right]. \quad (9)$$

If a temperature gradient exists the temperature can be considered to be constant over a section and equal to the mean temperature of the section. In this way the value of E is assumed to be constant over any one section. Δ then becomes proportional to Eh^3 and equation (8) can be written as

$$\delta S = \delta D = \left[\frac{E_2}{E_1}\left(\frac{h_2}{h_1}\right)^3 - 1\right]\left[\frac{S(1-\mu)+D(1+\mu)}{2}\right] \quad (10)$$

3. Edge Conditions

When using the charts for plates of varying thickness, two trial sets of S and D curves must be drawn. From these curves the correct initial values of S and D can be obtained, enabling the third and final solution to be drawn. The initial values of S and D for the two trials can be chosen arbitrarily but must in each case satisfy the boundary condition at the inner edge of the plate.

If the plate has no central hole, it will be evident from symmetry that the radial and tangential bending moments at the centre will be equal, hence

$$D = 0 \quad . \quad . \quad . \quad . \quad (11)$$

For a built-in edge the slope at the edge is zero. Hence, from equation (4),

$$0 = \frac{r(M_t - \mu M_r)}{(1-\mu^2)\Delta}.$$

Therefore $$M_t = \mu M_r$$

and $$D = \frac{-S(1-\mu)}{(1+\mu)} \quad . \quad . \quad . \quad . \quad (12)$$

For a free edge M_r is zero, therefore

$$\frac{S-D}{2} = 0 \text{ and } D = S \quad . \quad . \quad . \quad (13)$$

4. Derivation of S and D Curves

By considering separately the three different systems of loading, equations for S and D can be derived. Each equation contains an arbitrary constant and by assigning to it various values a family of curves can be drawn for each equation. Skeleton charts are given in Figs. 1 (*a*), (*b*), (*c*) and (*d*). In constructing the charts the dimensions should be as large as is convenient and the number of curves such that interpolation can be done visually to the degree of accuracy required. The scale of S and D values can be varied to cover likely requirements.

5. Concentrated Circumferential Load

The shear force per unit peripheral length $= Q = P/(2\pi r)$ where P = total load.

By integrating equation (1) twice,

$$\frac{d\omega}{dr} = -\phi = \frac{Pr}{8\pi\Delta}(2\log r - 1) - \frac{C_1 r}{2} - \frac{C_2}{r}, \quad (14)$$

and differentiating this

$$\frac{d^2\omega}{dr^2} = -\frac{d\phi}{dr} = \frac{P}{8\pi\Delta}(2\log r + 1) - \frac{C_1}{2} + \frac{C_2}{r^2}. \quad (15)$$

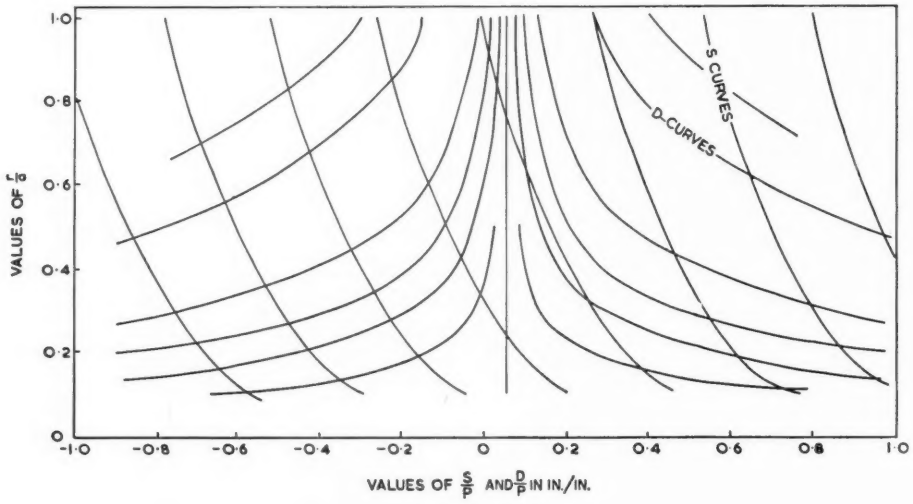

FIGURE 1(*a*). Bending moment chart for concentrated peripheral loading.

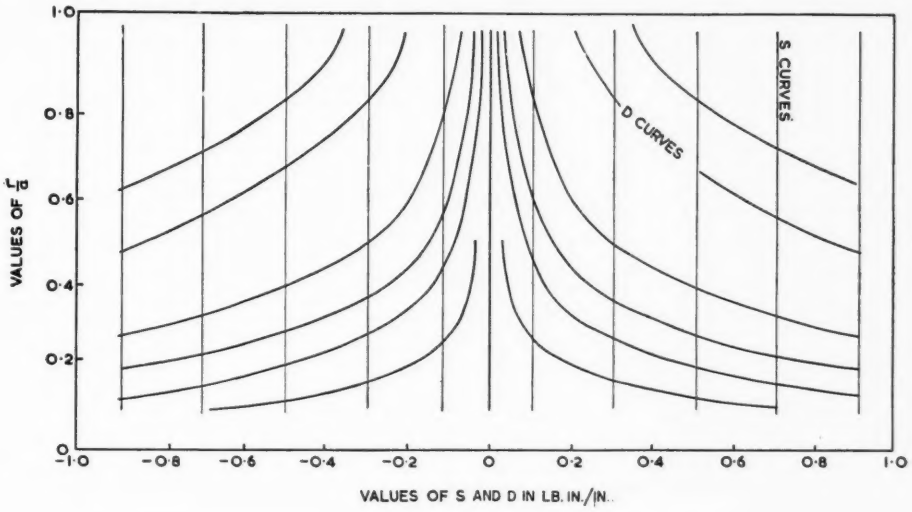

FIGURE 1(*b*). Bending moment chart for loading by applied couples.

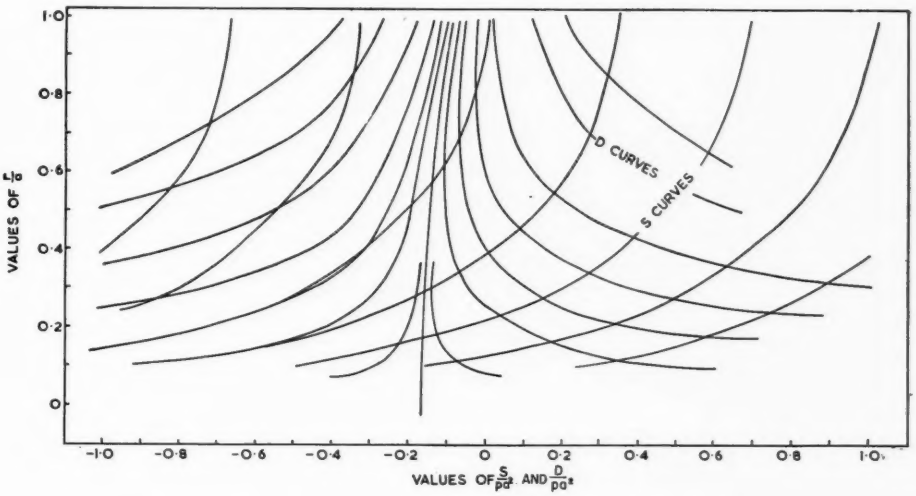

FIGURE 1(*c*). Bending moment chart. Pressure loading. Supported at inside edge.

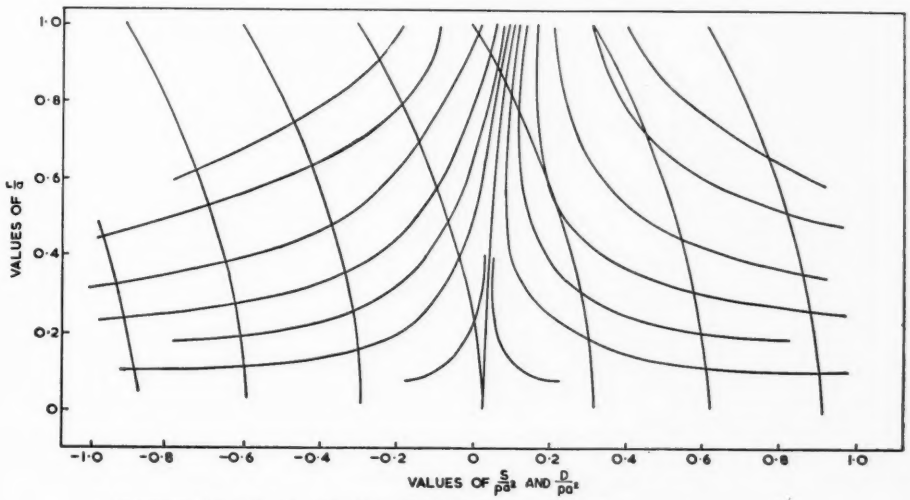

FIGURE 1(*d*). Bending moment chart. Pressure loading. No central hole.

FIGURES 1(*a*), (*b*), (*c*) and (*d*). Bending moment charts.
(These charts are not sufficiently accurate to allow scaling up).

Substituting (14) and (15) in equations (2) and (3) and simplifying,

$$M_r = -\Delta\left[\frac{P}{4\pi\Delta}(1+\mu)\log r + \frac{P}{8\pi\Delta}(1-\mu) - \frac{C_1}{2}(1+\mu) + \frac{C_2}{r^2}(1-\mu)\right] \quad . \quad . \quad (16)$$

$$M_t = -\Delta\left[\frac{P}{4\pi\Delta}(1+\mu)\log r - \frac{P}{8\pi\Delta}(1-\mu) - \frac{C_1}{2}(1+\mu) - \frac{C_2}{r^2}(1-\mu)\right] \quad . \quad . \quad (17)$$

Putting $S = M_t + M_r$ and $D = M_t - M_r$ into equations (16) and (17),

$$S = -\Delta\left[\frac{P}{2\pi\Delta}(1+\mu)\log r - C_1(1+\mu)\right]$$

$$D = -\Delta\left[-\frac{P}{4\pi\Delta}(1-\mu) - \frac{2C_2}{r^2}(1-\mu)\right].$$

For a constant thickness section Δ is constant. Hence, putting

$$C_1 = \frac{A_1 P}{\Delta} \text{ and } C_2 = \frac{B_1 P}{2\Delta},$$

gives

$$S = -P\left[\frac{(1+\mu)}{2\pi}\log r - A_1(1+\mu)\right]$$

$$D = P\left[\frac{(1-\mu)}{4\pi} + \frac{B_1}{r^2}(1+\mu)\right].$$

Putting the radius into the non-dimensional form $\rho = r/a$,

$$S = -P\left[\frac{(1+\mu)}{2\pi}\log\rho + \frac{(1+\mu)}{2\pi}\log a - A_1(1+\mu)\right],$$

and putting

$$\frac{\log a}{2\pi} - A_1 = A,$$

$$S = -P\left[\frac{(1+\mu)}{2\pi}\log\rho - (1+\mu)A\right] \quad . \quad (18)$$

$$D = P\left[\frac{(1-\mu)}{4\pi} + \frac{B_1}{a^2\rho^2}(1-\mu)\right] = P\left[\frac{(1-\mu)}{4\pi} + \frac{B}{\rho^2}(1-\mu)\right] \quad . \quad . \quad (19)$$

where $B_1 = Ba^2$.

6. Bending by Couples Applied at Edges

In this case the shear force is zero.

From equation (1),

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d\omega}{dr}\right)\right] = 0.$$

Integrating twice,

$$-\frac{d\omega}{dr} = \phi = \frac{C_1 r}{2} + \frac{C_2}{r} \quad . \quad . \quad (20)$$

and

$$-\frac{d^2\omega}{dr^2} = \frac{d\phi}{dr} = \frac{C_1}{2} - \frac{C_2}{r^2}. \quad . \quad (21)$$

Putting equations (20) and (21) in equations (2) and (3) and simplifying,

$$M_r = \Delta\left[\frac{C_1}{2}(1+\mu) - \frac{C_2}{r^2}(1-\mu)\right] \quad . \quad (22)$$

$$M_t = \Delta\left[\frac{C_1}{2}(1+\mu) + \frac{C_2}{r^2}(1-\mu)\right]. \quad . \quad (23)$$

For a constant thickness disc Δ is constant and equations (22) and (23) can be written as

$$M_r = A(1+\mu) - \frac{B_1}{r}(1-\mu)$$

$$M_t = A(1+\mu) + \frac{B_1}{r^2}(1-\mu).$$

Putting $S = M_t + M_r$ and $D = M_t - M_r$,

$$S = 2A_1(1+\mu)$$

$$D = \frac{2B_1}{r^2}(1-\mu).$$

Since $\rho = r/a$,

$$S = A(1+\mu) \quad . \quad . \quad . \quad . \quad (24)$$

$$D = \frac{B}{\rho^2}(1-\mu). \quad . \quad . \quad . \quad . \quad (25)$$

7. Uniformly Distributed Load

If the plate is supported at the inner radius

$$Q = \frac{p\pi(a^2 - r^2)}{2\pi r} = \frac{p}{2}\left(\frac{a^2}{r} - r\right). \quad . \quad (26)$$

and if supported at the outer radius

$$Q = \frac{p\pi(b^2 - r^2)}{2\pi r} = \frac{p}{2}\left(\frac{b^2}{r} - r\right).$$

Hence by substituting b for a in the final equations, both cases can be obtained.

Substituting equation (26) in equation (1) and integrating,

$$-\frac{d\omega}{dr} = \phi = \frac{pa^2 r}{8\Delta}(2\log r - 1) - \frac{pr^3}{16\Delta} + \frac{C_1 r}{2} + \frac{C_2}{r} \quad (27)$$

$$-\frac{d^2\omega}{dr^2} = \frac{d\phi}{dr} = \frac{pa^2}{8\Delta}(2\log r + 1) - \frac{3pr^2}{16\Delta} + \frac{C_1}{2} - \frac{C_2}{r^2} \quad (28)$$

Substituting equations (27) and (28) in (2) and (3) and simplifying,

$$M_r = \Delta\left[\frac{pa^2}{4\Delta}(1+\mu)\log r + \frac{pa^2}{8\Delta}(1-\mu) - \frac{pr^2}{16\Delta}(3+\mu) + \frac{C_1}{2}(1+\mu) - \frac{C_2}{r^2}(1-\mu)\right] \quad . \quad . \quad (29)$$

$$M_t = \Delta\left[\frac{pa^2}{4\Delta}(1+\mu)\log r - \frac{pa^2}{8\Delta}(1-\mu) - \frac{pr^2}{16\Delta}(1+3\mu) + \frac{C_1}{2}(1+\mu) + \frac{C_2}{r^2}(1-\mu)\right] \quad . \quad . \quad (30)$$

For a given plate Δ, a and p are constant when considering a constant thickness section. Hence, letting $C_1 = 2A_1 a^2 p/\Delta$ and $C_2 = B_1 a^2 p/\Delta$ and combining these with equations (29) and (30), presenting the result in sum and difference form,

$$S = pa^2\left[\frac{1+\mu}{2}\log r - \frac{r^2}{4a^2}(1+\mu) + 2A_1(1+\mu)\right] \quad (31)$$

$$D = pa^2\left[-\frac{1-\mu}{4} + \frac{r^2}{8a^2}(1-\mu) + \frac{2B_1}{r^2}(1-\mu)\right] \quad (32)$$

Expressing r in the non-dimensional form $\rho = r/a$ and adjusting the constants, then for a plate supported at the inner radius,

$$S = pa^2\left[\frac{1+\mu}{2}\log\rho - \rho^2\frac{(1+\mu)}{4} + A(1+\mu)\right] \quad (33)$$

$$D = pa^2\left[-\frac{(1-\mu)}{4} + \rho^2\frac{(1-\mu)}{8} + \frac{B}{\rho^2}(1-\mu)\right] \quad (34)$$

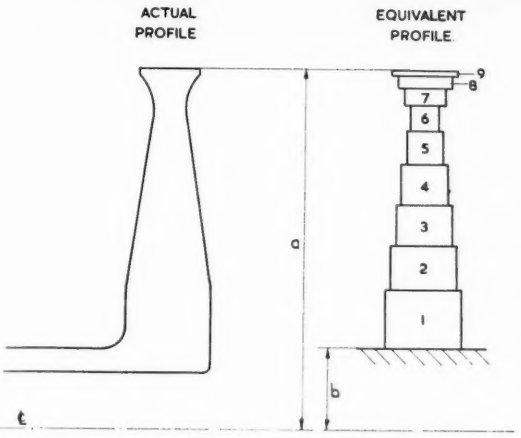

FIGURE 2. Division of a disc profile into an equivalent system of constant thickness annuli.

and for a plate supported at the outer radius,

$$S=pb^2\left[\frac{1+\mu}{2}\log\lambda-\frac{\lambda^2(1+\mu)}{4}+A(1+\mu)\right] \quad (35)$$

$$D=pb^2\left[-\frac{1-\mu}{4}+\frac{\lambda^2(1-\mu)}{8}+\frac{B}{\lambda^2}(1-\mu)\right]. \quad (36)$$

A plate supported at the outside edge with central hole and pressure loading is not likely to be met with in practice. The more common case, of support at the outside edge with no central hole, cannot be dealt with by means of equations (35) and (36), since when $b=0$ the equations take the form $S=0\times\infty$ and $D=0\times\infty$. Hence, for this case a separate chart will be needed. The derivation of the S and D curves is as follows:—

From equations (29) and (30), substituting b for a and writing $b=0$,

$$S=\Delta\left[-\frac{pr^2}{4\Delta}(1+\mu)+C_1(1+\mu)\right] \quad (37)$$

$$D=\Delta\left[\frac{pr^2}{8\Delta}(1-\mu)+\frac{2C_2}{r^2}(1-\mu)\right] \quad (38)$$

Over a constant thickness section Δ and p remain constant and equations (37) and (38) can be written as

$$S=p\left[-\frac{r^2}{4}(1+\mu)+A_1(1+\mu)\right]$$

$$D=p\left[\frac{r^2}{8}(1-\mu)+\frac{B_1}{r^2}(1-\mu)\right].$$

Expressing r in terms of ρ,

$$S=pa^2\left[-\frac{\rho^2(1+\mu)}{4}+A(1+\mu)\right] \quad (39)$$

$$D=pa^2\left[\frac{\rho^2(1-\mu)}{8}+\frac{B(1-\mu)}{\rho^2}\right]. \quad (40)$$

8. Using the Chart

For a constant thickness disc with no temperature gradient the solution consists of finding a pair of S and D curves satisfying the boundary conditions at the inner and outer radii.

For a plate of varying thickness the profile should be divided into constant thickness steps (Fig. 2) and a tabulation made of the radius of the section changes, r/a (or r/b for pressure loading with support at outer radius) h, $(h_2/h_1)^3-1$, E, and E_2/E_1.

Tracing paper should then be placed over the chart with horizontal lines drawn at the various values of r. Beginning at r_0, the inside radius, a pair of S and D curves can be drawn, remembering that the values of S and D at r_0 must satisfy equation (11), (12) or (13) according to the boundary conditions at the inner edge. At the termination of the first section the values of S and D arrived at can be read from the chart, the change in S and D being computed from equation (10) and the values of δS and δD marked off along the horizontal line, due notice being taken of the signs. Beginning at the new values of S and D the contour of the S and D curves can then be followed again until the boundary of the second section is reached. This procedure is followed up to the outer edge of the plate (Fig. 3). A second set of curves must now be drawn, choosing a different value for S and D at the inner radius. By substituting the results in two equations of the form

$$M_{t(\text{bore})}=X+YM_{r(\text{rim})}$$

the values of X and Y can be computed. Hence, knowing these and the required value of $M_{r(\text{rim})}$, $M_{t(\text{bore})}$ can be determined and a third and final set of S and D curves obtained.

8.1. STRESSES

The bending moments obtained from the chart can be used to find the stress at any radius, the mean value of M_r and M_t at each section being plotted against the mean radius of the sections.

The stress is given by

$$\sigma_r=\frac{6M_r}{h^2}$$

$$\sigma_t=\frac{6M_t}{h^2}.$$

8.2. DEFLECTIONS

Knowing the values of S and D at any radius, the slope at any radius can be obtained from equation (5).

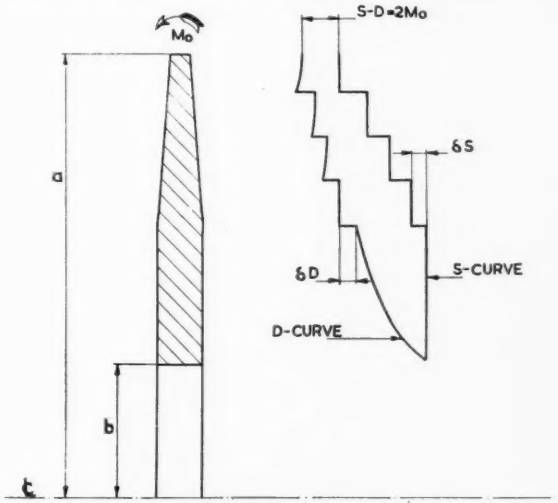

FIGURE 3. An example showing the form of the S and D curves for the plate and type of loading illustrated.

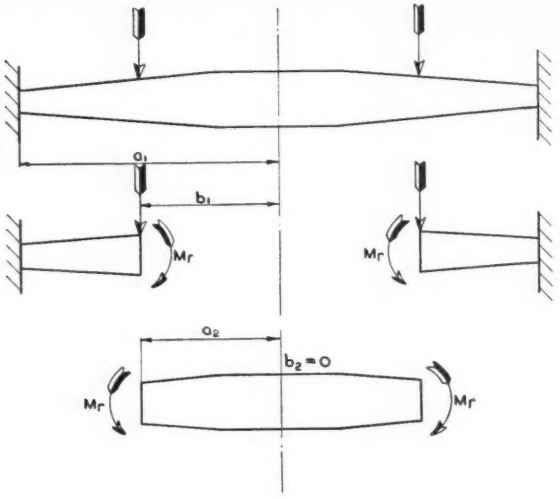

FIGURE 4. Simplifying a compound system of loading into two cases of simple loading.

By plotting the values of slope against radius and integrating the diagram graphically, the deflection of the plate can be found.

9. Concentric Loads Applied with the Edges of the Plate

When the load is applied concentrically but not at the edges of the plate, the problem can be overcome by suitably dividing the disc, solving each section separately and equating the radial bending moments at the junction. An example of this is given in Fig. 4. The centre disc and the annulus are worked separately on the appropriate charts, the values of M_r obtained at the outer rim of the disc in each trial being used as the initial values of M_r at the inner edge of the annulus. Since the slopes at the junction of the edges must also be equal, δS and δD across the junction are obtained from equation (9).

10. Limitations of Method

The assumptions on which this method is based are those normally taken in the simple theory of plates and shells, and therefore the method is subject to the same limitations. The main assumptions are that the deflections are small in comparison with the thickness and that the ratio of thickness to outside diameter is also small. Further information on this point can be obtained from Ref. 1, where it will be seen that the errors introduced by this assumption vary with the nature of the plate loading. It is possible however to generalise to some extent and the theory may be taken to hold if the deflection does not exceed 0·4 to 0·5 of the plate mean thickness. Similarly the error introduced by the thickness of the plate may be taken to be of the order $1{\cdot}1\,(h/a)^2$.

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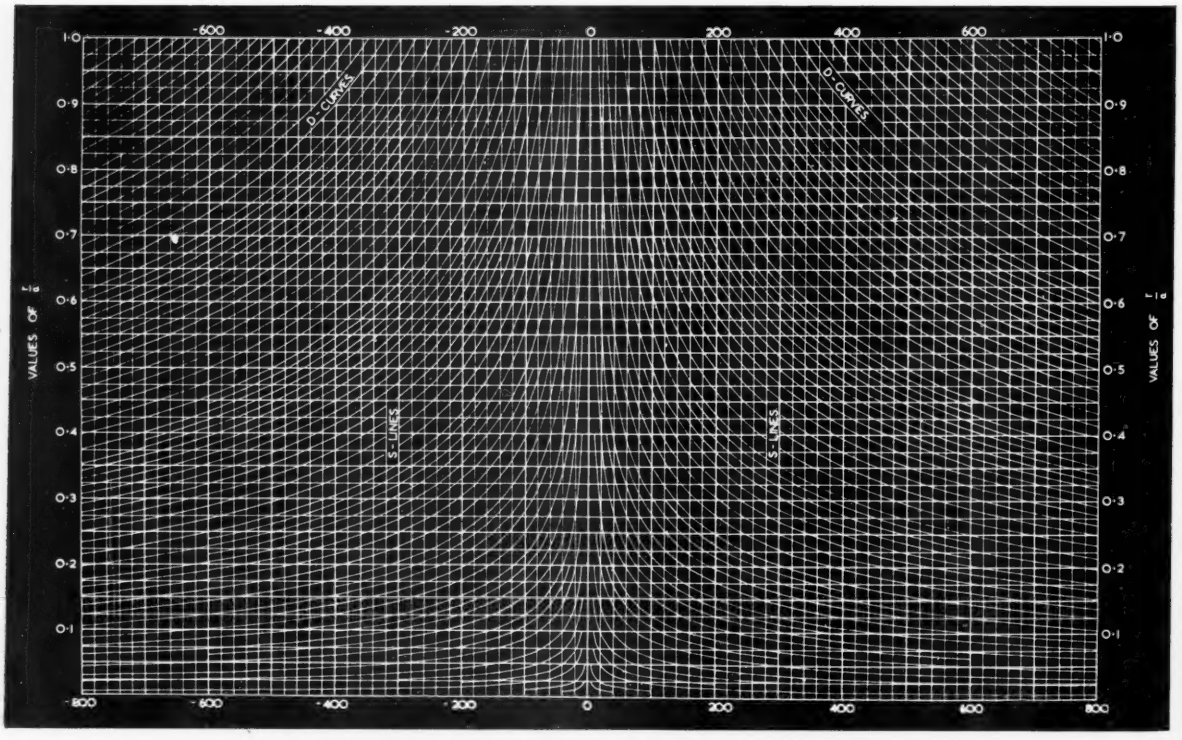

FIGURE 5. Bending moment chart for loading by applied couples.

Normal Shock Wave Phenomena in a Convergent-Divergent Nozzle

by

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SUMMARY:—In recent years, aerodynamics and thermodynamics have found common ground in the specialised field of gas dynamics. Developments in this subject have led to a much more complete and widespread knowledge of subsonic, sonic and supersonic flow of gases in the conventional type of convergent-divergent nozzle. When the back pressure is raised above the value against which the nozzle is designed to discharge, oblique and then normal shock waves are set up in the divergent cone at a position along the nozzle axis determined by the magnitude of that back pressure. The gas which has crossed the shock wave is subjected to a process of subsonic compression.

In this paper a theoretical investigation is made of the changes in pressure, temperature, density and Mach number which occur across a normal shock wave, when the position of the wave varies along the nozzle axis. The investigation illustrates the effect of change of medium, for which the relevant property is the ratio of specific heats. This ratio for certain polyatomic gases may approach unity (*e.g.* for Dichlorodifluoromethane CCl_2F_2, in gaseous form, $\gamma = 1{\cdot}06$), and for the inert monatomic gases $\gamma = 1{\cdot}667$. The analysis is made non-dimensional by expressing such quantities as gas pressure, temperature, and density at any given position along the nozzle axis relative to the values of the particular parameter at entry to the convergent section.

1. Introduction

It is well known that for frictionless adiabatic expansion of a gas in a convergent-divergent nozzle, of the De Laval type, correctly designed for the back pressure against which the gas is finally discharged, the static pressure ratio plotted against nozzle length will have the form shown in Fig. 1. The critical pressure ratio will exist at the throat section "2," given by

$$\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

Supersonic flow will occur in the divergent length of nozzle.

If the back pressure is gradually increased beyond the design value, oblique shock waves form at the mouth of the nozzle, to be followed by a normal shock wave which advances up the nozzle towards the throat[1]. As the wave reaches the throat it disappears, and a further increase in back pressure reduces the nozzle to a simple venturi tube, so that supersonic flow no longer occurs. The magnitude of the theoretically instantaneous pressure increase which accompanies the normal shock wave will vary with the position of the shock wave in the nozzle, while the flow process following such a wave will be one of subsonic compression. It is natural that a shock wave should give rise to a sudden increase in entropy, illustrating the thermodynamically wasteful nature of the process. There will be also simultaneous changes of density, velocity, temperature and Mach number. The magnitude of all these changes will depend not only on the position of the shock wave, but also on the nature of the medium involved, represented by the ratio of the two specific heats.

The experimental results of Stodola[2] indicate that while the extent of the pressure changes consequent upon a shock wave accord fairly well with those predicted theoretically, a finite interval of time is required in practice for the build-up in pressure. This would seem to raise the question of the thickness of the shock wave within the nozzle, and also, the influence of the boundary layer, but these points are outside the scope of the present paper.

It is not proposed to specify the profile of the divergent cone, and therefore it is convenient to relate the pressure, or other loci, not to the position of the shock wave along the axis of the nozzle, but to the pressure, or other ratio, which would obtain in the absence of such a wave. Given the profile, the former relationship may then be established.

Notation

p	static pressure
ρ	density
T	absolute temperature
A	normal area of flow path
R	pressure ratio, *e.g.* $R_x = p_x/p_1$
C.P.R.	critical pressure ratio
V	velocity of flow
M	Mach number
γ	specific heat ratio

Suffixes "x" and "s" denote conditions immediately before and after a normal shock wave respectively. Suffix "c" refers to the critical condition which occurs

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at the throat of the nozzle. Suffixes 1, 2 refer to the positions shown in Fig. 1.

2. Pressure Locus following a Normal Shock Wave

From considerations of conservation of mass, energy and momentum, it may be shown that if there is a shock wave at any section (Fig. 1) in the divergent length of nozzle, the pressure ratio across the wave is given by

$$\frac{p_s}{p_x}=\frac{2\gamma M_x^2-\gamma+1}{\gamma+1},$$

while the Mach number before the wave is

$$M_x=\left[\frac{2}{\gamma-1}\left\{\left(\frac{p_1}{p_x}\right)^{\frac{\gamma-1}{\gamma}}-1\right\}\right]^{\frac{1}{2}}.$$

Elimination of M_x yields the equation

$$\frac{p_s}{p_x}(\gamma^2-1)+(\gamma+1)^2=4\gamma\left(\frac{p_1}{p_x}\right)^{\frac{\gamma-1}{\gamma}}$$

or

$$\frac{p_s}{p_1}(\gamma^2-1)+\frac{p_x}{p_1}(\gamma+1)^2=4\gamma\left(\frac{p_x}{p_1}\right)^{\frac{1}{\gamma}}.$$

If

$$R_s=\frac{p_s}{p_1} \text{ and } R_x=\frac{p_x}{p_1},$$

the previous equation becomes

$$R_s=\frac{4\gamma R_x^{1/\gamma}-R_x(\gamma+1)^2}{\gamma^2-1} \quad . \quad . \quad (1)$$

An alternative form is

$$R_s=\frac{R_x(2\gamma M_x^2-\gamma+1)}{\gamma+1} \quad . \quad . \quad (2)$$

This relationship is such that

if $R_x=0,\ R_s=R_x$

and $R_s=R_{xc}=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

when $M_x=1$.

These two conditions hold good at the mouth of an infinitely long nozzle discharging to a vacuum, and at the throat respectively. Since M_x increases as R_x decreases, there will be a maximum value of R_s somewhere between these limits. It is given by

$$\frac{dR_s}{dR_x}=\frac{4R_x^{\frac{1-\gamma}{\gamma}}-(\gamma+1)^2}{\gamma^2-1}=0$$

so that

$$R_x=\left(\frac{2}{\gamma+1}\right)^{\frac{2\gamma}{\gamma-1}}=R_{xc}^2 \quad . \quad . \quad (3)$$

Thus the shock wave pressure locus has its maximum value when the pressure ratio just before the shock wave is the square of the critical pressure ratio.

$$\text{Maximum } R_s=(\gamma+1)\left(\frac{2}{\gamma+1}\right)^{\frac{2\gamma}{\gamma-1}}=(\gamma+1)R_{xc}^2, \quad (4)$$

which is obtained by substituting for (3) in (1). The function represented by equation (1) may now be plotted

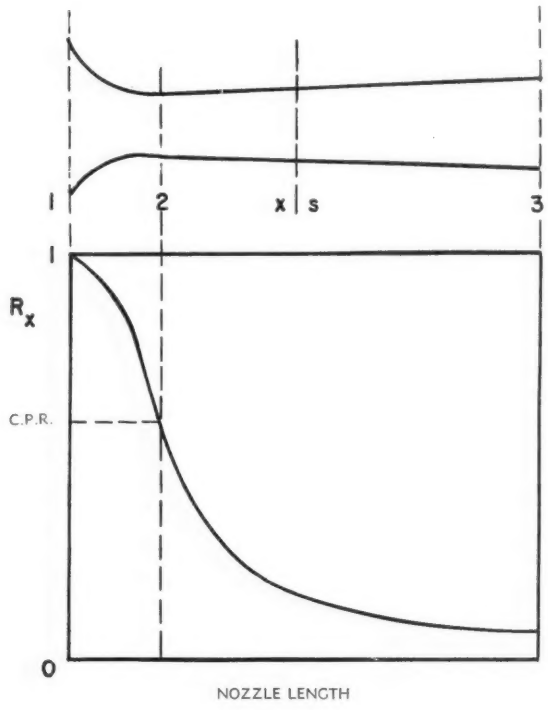

FIGURE 1.

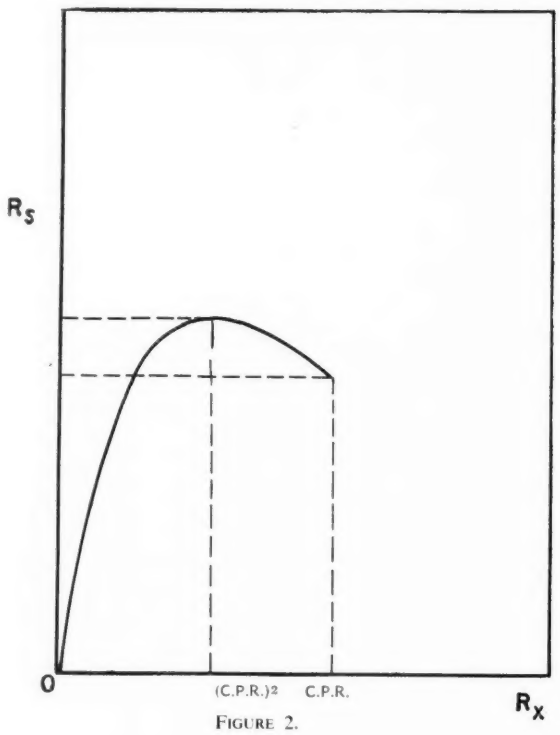

FIGURE 2.

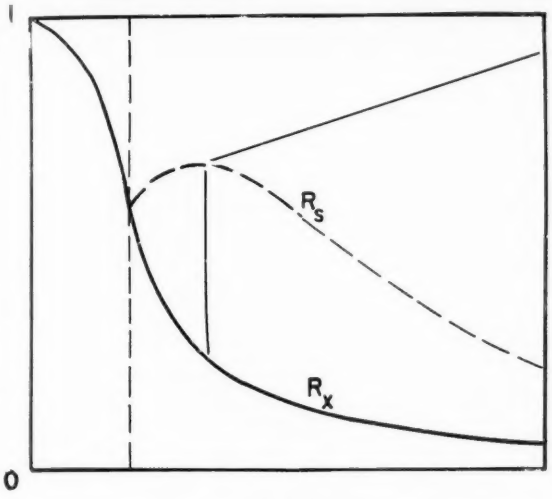

FIGURE 3.

(Fig. 2). The shock pressure plotted on an abscissa of nozzle length has the general form shown by the dotted line in Fig. 3.

The functions of γ already derived for the square of the critical pressure ratio and the maximum shock pressure are such that they decrease as γ increases. Their values are, however, indeterminate, in the limiting case when γ tends to unity, and must be found by logarithmic differentiation. In each case they are simple functions of e; thus when $\gamma \longrightarrow 1$,

$$\frac{2}{\gamma+1}^{\frac{2\gamma}{\gamma-1}} \longrightarrow \frac{1}{e} \text{ and } (\gamma+1)\left(\frac{2}{\gamma+1}\right)^{\frac{2\gamma}{\gamma-1}} \longrightarrow \frac{2}{e}.$$

It is of interest to determine values of Mach number, density, temperature, and area of flow path for the condition of maximum shock wave pressure.

$$M_x = \left[\frac{2}{\gamma-1}\left\{\left(\frac{p_1}{p_x}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\}\right]^{\frac{1}{2}} \text{ where } \frac{p_x}{p_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{2\gamma}{\gamma-1}}$$

whence
$$M_x = \left(\frac{\gamma+3}{2}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad (5)$$

The Mach numbers before and after a normal shock wave are connected by the relationship

$$M_s^2 = \frac{1+\left(\frac{\gamma-1}{2}\right)M_x^2}{\gamma M_x^2 - \left(\frac{\gamma-1}{2}\right)}.$$

Substituting from (5),

$$M_s = 2^{-\frac{1}{2}} \simeq 0{\cdot}707.$$

The Mach number behind a normal shock wave for the condition of maximum shock wave pressure is therefore independent of the medium flowing through the nozzle.

The density ratio $\frac{\rho_s}{\rho_x} = \frac{(\gamma+1)M_x^2}{2+(\gamma-1)M_x^2}$.

Using (5) again, $\frac{\rho_s}{\rho_x} = \frac{\gamma+3}{\gamma+1}$ . . . . . (6)

so that
$$\frac{\rho_s}{\rho_1} = \frac{\rho_s}{\rho_x}\frac{\rho_x}{\rho_1} = \left(\frac{\gamma+3}{\gamma+1}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} \quad . \quad (7)$$

It will be seen later that this expression does not represent the condition of maximum density after a normal shock wave, as might be thought, though by so assuming no great error is introduced. The function increases slightly with specific heat ratio, varying between 0·736 and 0·739 over the range considered.

From the principle of continuity of flow, equation (6) must express also the velocity ratio across the shock wave for the condition of maximum shock wave pressure,

i.e.
$$\frac{\rho_s}{\rho_x} = \frac{V_x}{V_s} = \frac{\gamma+3}{\gamma+1}.$$

The temperature ratio

$$\frac{T_s}{T_x} = \frac{p_s}{p_x}\frac{\rho_x}{\rho_s} = \frac{(\gamma+1)^2}{\gamma+3}$$

and
$$\frac{T_s}{T_1} = \frac{T_s}{T_x}\frac{T_x}{T_1} = \frac{(\gamma+1)^2}{\gamma+3}\left(\frac{2}{\gamma+1}\right)^2 = \frac{4}{\gamma+3} \quad . \quad (8)$$

which function varies inversely with γ.

The normal area of flow expressed as a proportion of the throat area is

$$\frac{A_x}{A_2} = \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\left(\frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{2}}}{\left[\left(\frac{p_x}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_x}{p_1}\right)^{\frac{\gamma+1}{\gamma}}\right]^{\frac{1}{2}}}$$

and
$$\frac{p_x}{p_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{2\gamma}{\gamma-1}}$$

giving
$$\frac{A_x}{A_2} = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}}\left(\frac{\gamma+1}{\gamma+3}\right)^{\frac{1}{2}} \quad . \quad . \quad (9)$$

which has a maximum of $(e/2)^{\frac{1}{2}}$ or 1·1658 as $\gamma \longrightarrow 1$, and falls to 1·1635 when $\gamma = 5/3$. This represents a change in area of less than 0·2 per cent., so for all practical purposes the condition of maximum pressure after a normal shock wave occurs when the area of flow is approximately 16·5 per cent. greater than that at the throat, no matter what the medium may be.

3. Density Locus following a Normal Shock Wave

Referring again to Fig. 1, it may be shown that the density ratio across the shock wave is

$$\frac{\rho_s}{\rho_x} = \frac{(\gamma+1)M_x^2}{2+(\gamma-1)M_x^2},$$

while

$$M_x^2 = \frac{2}{\gamma-1}\left[\left(\frac{\rho_1}{\rho_x}\right)^{\gamma-1} - 1\right].$$

so that

$$\frac{\rho_s}{\rho_1} = \frac{\rho_s}{\rho_x}\frac{\rho_x}{\rho_1} = \left(\frac{\gamma+1}{\gamma-1}\right)\left[\left(\frac{\rho_x}{\rho_1}\right) - \left(\frac{\rho_x}{\rho_1}\right)^{\gamma}\right]. \quad (10)$$

The non-dimensional shock wave density therefore depends on the Mach number and the density which exists before the shock wave. As the former increases, the latter decreases, leading to a maximum value of equation (10) given by

$$\frac{\rho_x}{\rho_1} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}} \quad . \quad . \quad . \quad . \quad (11)$$

or

$$\frac{p_x}{p_1} = \left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}.$$

The expression obtained in equation (11) also decreases as γ increases, but at a lower rate than that of equation (3). Both expressions have the same numerical value as $\gamma \longrightarrow 1$. It follows that the condition for maximum shock wave density occurs at a higher pre-shock wave pressure ratio and a smaller area ratio than that for maximum shock wave pressure, and the deviation between the two conditions increases with γ.

Substitution of (11) in (10) yields

$$\text{Maximum} \quad \frac{\rho_s}{\rho_1} = \frac{\gamma+1}{(\gamma)^{\frac{\gamma}{\gamma-1}}} \quad . \quad . \quad . \quad (12)$$

This function of γ, which varies between 0·736 and 0·744 increases rather more rapidly than the function of equation (7).

The general form of equation (10) is otherwise the same as that shown in Fig 3 for the pressure locus. The Mach numbers associated with maximum shock wave density are found as before, so that

$$M_x^2 = \left(\frac{2}{\gamma-1}\right)(\gamma-1) \text{ and } M_x \simeq 1{\cdot}414.$$

In this case the Mach number before the normal shock wave under conditions of maximum shock wave density is not a function of the medium.

$$M_s^2 = \frac{1+\left(\frac{\gamma-1}{2}\right)M_x^2}{\gamma M_x^2 - \frac{(\gamma-1)}{2}},$$

and $M_s = \left(\frac{2}{3\gamma+1}\right)^{\frac{1}{2}}$ which increases with γ.

For the same condition

$$R_s = \left(\frac{3\gamma+1}{\gamma+1}\right)\frac{1}{(\gamma)^{\frac{\gamma}{\gamma-1}}},$$

which has slightly lower values than those of equation (4).

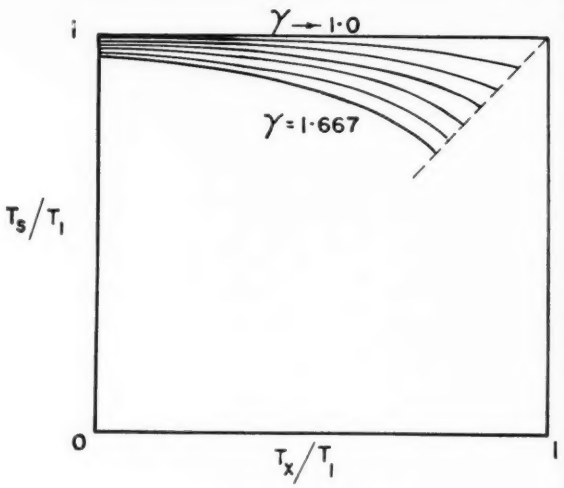

FIGURE 4.

The temperature ratio

$$\frac{T_s}{T_x} = \frac{p_s}{p_x}\frac{\rho_x}{\rho_s} = \frac{\gamma(3\gamma+1)}{(\gamma+1)^2}$$

$$\frac{T_s}{T_1} = \frac{T_s}{T_x}\frac{T_x}{T_1} = \frac{3\gamma+1}{(\gamma+1)^2},$$

which decreases with increasing γ.

Finally

$$\frac{A_x}{A_2} = \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\left(\frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{2}}}{\left[\left(\frac{\rho_x}{\rho_1}\right)^2 - \left(\frac{\rho_x}{\rho_1}\right)^{\gamma+1}\right]^{\frac{1}{2}}},$$

where $\frac{\rho_x}{\rho_1} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}$,

and hence $\frac{A_x}{A_2} = \left[2\left(\frac{\gamma}{\gamma+1}\right)^{\frac{\gamma+1}{2}}\right]^{\frac{1}{\gamma-1}}$. (13)

Unlike the area ratio of equation (9) there is in this case no maximum value, though as $\gamma \longrightarrow 1$ the function approaches $(e/2)^{\frac{1}{2}}$. For $\gamma = 5/3$, the area ratio is 1·102, representing a reduction of $5\frac{1}{2}$ per cent. The area ratio cannot be considered independent of the medium in this case.

4. Temperature Locus following a Normal Shock Wave

It can be shown that the temperature ratio across a normal shock wave is represented by the equation

$$\frac{T_s}{T_x} = \left[\frac{2\gamma M_x^2 - \gamma + 1}{\gamma+1}\right]\left[\frac{2+M_x^2(\gamma-1)}{(\gamma+1)M_x^2}\right]$$

and $M_x^2 = \left(\frac{2}{\gamma-1}\right)\left(\frac{T_1}{T_x} - 1\right)$,

the second equation expressing the pre-shock wave Mach number in terms of temperature. Eliminating M_x gives

$$\frac{T_s}{T_x}=\left[\frac{T_1}{T_1-T_x}\right]\left[\frac{4\gamma}{(\gamma+1)^2}\frac{T_1}{T_x}-1\right]$$

and

$$\frac{T_s}{T_1}=\frac{\dfrac{4\gamma}{(\gamma+1)^2}-\dfrac{T_x}{T_1}}{1-\dfrac{T_x}{T_1}} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

This relationship is shown in Fig. 4. It will be seen that T_s/T_1 approaches a limiting value of $4\gamma/(\gamma+1)^2$ as T_x/T_1 tends to zero. This limiting value, which is academic, decreases as γ increases and has a least possible value of 0·9375 when $\gamma=5/3$.

5. Mach Number following a Normal Shock Wave

Expressing shock wave Mach number in terms of the pressure ratio before the shock.

$$M_s^2=\frac{2(\gamma-1)}{4\gamma-(\gamma+1)^2R_x^{\frac{\gamma-1}{\gamma}}}.$$

As $R_x\longrightarrow 0,\ M_s\longrightarrow\left(\dfrac{\gamma-1}{2\gamma}\right)^{\frac{1}{2}}$.

The Mach number following a shock wave at the mouth of an infinitely long nozzle therefore increases with the ratio of specific heats.

When $\gamma=5/3,\ M_s=0{\cdot}447$.

6. Maximum Pressure and Density Jump across a Normal Shock Wave

It has been shown that

$$R_s(\gamma^2-1)+R_x(\gamma+1)^2=4\gamma R_x^{\frac{1}{\gamma}},$$

and hence the pressure jump ΔR across the shock wave is

$$\Delta R=R_s-R_x=\frac{2\gamma}{\gamma^2-1}\left[2R_x^{\frac{1}{\gamma}}-R_x(\gamma+1)\right]. \quad (15)$$

Differentiation of (15) yields a turning point which from Fig. 3 must be a maximum. It occurs when

$$R_x=\left[\frac{2}{\gamma(\gamma+1)}\right]^{\frac{\gamma}{\gamma-1}} \quad . \quad . \quad . \quad (16)$$

which is, of course, not the same as equation (3). Equation (16) tends to a value of $e^{-\frac{3}{2}}$ as γ tends to unity and decreases with increasing values of γ. Substitution of the expression of equation (16) in equation (15) gives

$$\text{Maximum } \Delta R=\left[\frac{2^{2\gamma-1}}{\gamma(\gamma+1)^\gamma}\right]^{\frac{1}{\gamma-1}} \quad . \quad (17)$$

Logarithmic differentiation of equation (17) shows that it has a maximum value obtained by the value of γ which satisfies the equation

$$\frac{\gamma(\gamma+1)}{2}=\exp\left\{\frac{(\gamma^2+\gamma+1)(\gamma-1)}{\gamma(\gamma+1)}\right\}$$

a solution of which is $\gamma\simeq 2^{\frac{1}{2}}=1{\cdot}414$. Then the maximum value of the pressure jump is 0·456.

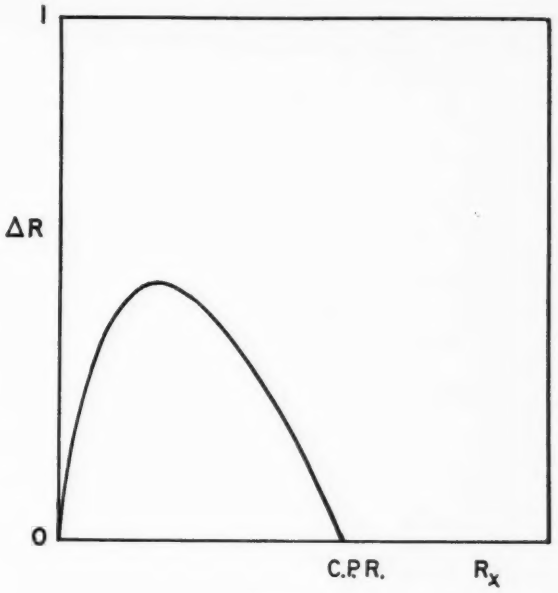

FIGURE 5.

Other values are

$$\gamma\longrightarrow 1,\ \Delta R\longrightarrow 2e^{-\frac{3}{2}}=0{\cdot}447.$$

$$\gamma=5/3,\ \Delta R=0{\cdot}453.$$

While the greatest possible percentage change due to change of medium is thus only of the order of 2, it is nevertheless clear that diatomic gases, for which $\gamma\simeq 1{\cdot}4$, give rise to the maximum possible pressure jump.

The pressure jump is shown plotted against pre-shock pressure ratio in Fig. 5.

The non-dimensional density jump is

$$\frac{\rho_s-\rho_x}{\rho_1}=\left(\frac{\gamma+1}{\gamma-1}\right)\left[\left(\frac{\rho_x}{\rho_1}\right)-\left(\frac{\rho_x}{\rho_1}\right)^\gamma\right]-\frac{\rho_x}{\rho_1}.$$

This is greatest when

$$\frac{\rho_x}{\rho_1}=\left[\frac{2}{\gamma(\gamma+1)}\right]^{\frac{\gamma}{\gamma-1}}$$

i.e. when $\dfrac{p_x}{p_1}=\left[\dfrac{2}{\gamma(\gamma+1)}\right]^{\frac{\gamma}{\gamma-1}}$.

This is the case just investigated and so the maximum pressure and maximum density jumps occur simultaneously. It is found that the maximum density jump is

$$\left[\frac{1}{(\gamma+1)}\left(\frac{2}{\gamma}\right)^\gamma\right]^{\frac{1}{\gamma-1}} \quad . \quad . \quad . \quad (18)$$

which tends towards $2e^{-\frac{3}{2}}$ as γ tends to unity.

For this case

$$M_x=(\gamma+2)^{\frac{1}{2}}$$

$$M_s=\left(\frac{\gamma}{2\gamma+1}\right)^{\frac{1}{2}},\ \text{increasing with } \gamma.$$

Also $$\frac{T_s}{T_x} = \frac{\gamma(2\gamma+1)}{\gamma+2},$$

and $$\frac{T_s}{T_1} = \frac{2(2\gamma+1)}{(\gamma+1)(\gamma+2)}.$$

The non-dimensional area is found, after simplification, to be for this case

$$\frac{A_x}{A_2} = \left[\frac{\gamma^{\frac{\gamma+1}{\gamma-1}}}{\gamma+2}\right]^{\frac{1}{2}}. \qquad (19)$$

When $\gamma \longrightarrow 1$, $\frac{A_x}{A_2} \longrightarrow \frac{e}{3^{\frac{1}{2}}} = 1{\cdot}569$,

$\gamma = 5/3$, $\frac{A_x}{A_2} = 1{\cdot}450$.

The section of the nozzle for maximum jump values of density and pressure is therefore always downstream of the section at which maximum shock pressure occurs. It depends on the specific heat ratio whether the latter is substantially downstream of the section at which maximum shock density occurs.

7. Conclusions

From the results obtained, the ideal loci of shock pressure, shock density and shock temperature in a convergent-divergent nozzle may be readily plotted. The nature of these loci indicate that there are points of inflexion which, however, are not readily susceptible to analysis. For the pressure locus, for instance, it can be shown that the condition for a point of inflexion is

$$\frac{d^2}{dR_x^2}\left(\frac{A_x}{A_2}\right) = \frac{d}{dR_x}\left(\frac{A_x}{A_2}\right)\frac{dR_x}{dR_x}\frac{d^2R_x}{dR_x^2},$$

leading eventually to

$$4(\gamma^3+\gamma^2+13\gamma+9)R_x^{\frac{4-2\gamma}{\gamma}} + (3\gamma^4+14\gamma^3+24\gamma^2+18\gamma+5)R_x^{\frac{2}{\gamma}} = 32R_x^{\frac{5-3\gamma}{\gamma}} + 2(7\gamma^3+25\gamma^2+27\gamma+9)R_x^{\frac{3-\gamma}{\gamma}}.$$

This can be solved for R_x only when γ is known.

It is worthy of note that the shock pressure locus falls below the initial critical pressure for a value of R_x determined by the alternative root of

$$R_x - \frac{4\gamma}{(\gamma+1)^2}R_x^{\frac{1}{\gamma}} + \frac{2^{\frac{\gamma}{\gamma-1}}(\gamma-1)}{(\gamma+1)^{\frac{2\gamma-1}{\gamma-1}}} = 0.$$

This second root (Fig. 2) can be evaluated only on a purely numerical basis by substituting particular values of γ. The difference between the two roots is found to be approximately 0·426 over the working range of γ. The nozzle area where the shock pressure is equal to the critical pressure is about 91 per cent. greater than the throat area over the range of γ under consideration. For all areas greater than this, obtained by moving downstream, the shock wave pressure ratio is less than the critical pressure ratio. Yet the flow after the shock wave is subsonic. Pressure ratios below the critical ratio in a convergent-divergent nozzle are therefore not always necessarily synonymous with supersonic flow.

ACKNOWLEDGMENT

The author wishes to express his thanks to Professor A. F. Burstall, of the University of Durham, for his advice and encouragement in the preparation of the paper.

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Integral Construction

Report of the Discussion Following The Section Lecture "The Application of Integral Construction to Aircraft Design and Its Effect on Production Methods" by E. D. Keen given on 12th February 1953. The rapporteur for the Discussion was K. L. C. Legg. The complete paper was published in the April 1953 Journal.

MR JAMES (Vickers-Armstrongs Ltd.): He thought there was a possibility of considerable distortion during the machining and heat treatment operations. He quoted one example of the released stresses being sufficient to fracture the holding down bolts when machining a large component in the solution treated condition. It appeared that the suction beds which were necessary for machining thick plates of large area in the manner envisaged would have to work at near vacuum conditions if accurate machining to thicknesses of the order of 0·15 in. or less were to be possible.

He also questioned the availability of an adequate supply of machines for manufacture by integral construction methods to keep pace with actual aircraft production needs, particularly under war-time conditions.

MR KEEN: He agreed that distortion was a very real problem. His firm's experience on the tailplane project was that the sequence of machining operations and heat treatments were of considerable importance and it appeared that each type of integral construction project must, at the moment, be treated individually.

Ordinary spar millers could be used for most of the machining and there should be no hold-up there. A possible source of bottle neck was the use of a Hydrotel machine, but this was not necessarily the ultimate method of producing the profile; he considered stretch-forming to be the real answer.

MR. LEGG (Short Bros. and Harland Ltd.): His firm were concerned with the construction and testing of a spar box, under Ministry contract, of an existing aircraft, the upper and lower surfaces of which were integrally machined. These planks were machined from stretch levelled solid sheet approximately 22 in. wide by 2½ in. thick and 10 ft. long in D.T.D. 646 material. The typical section employed was as shown, the dimensions, b, d, t_s and t varying as a straight taper along the span. An analysis on the lines of a paper by Farrar on optimum conditions in compression* showed that values of 0·80 to 0·85 of the efficiency parameter $\sqrt{\{f^2 I/(P E_T)\}}$ could be achieved. It was favourable to choose the lowest possible ratio of stringer area/skin area consistent with efficient design as this would provide the greatest skin thickness and, therefore, the greatest torsional stiffness. Preliminary panel tests on this form of construction had realised compressive stresses of the order of 60,000 lb./in.², these high stresses being largely due to the fixation effect at the solid "T" junction.

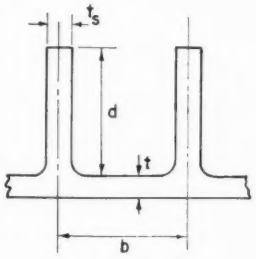

FIGURE 1.

The spar box planks were contoured on a planing machine and the webs (and skin) were taper machined on a spar miller. A suction bed was used and it was found that adequate suction was available employing a normal suction pump of the plunger type. The order of skin and web thicknesses was 0·20 in. at the root and 0·06 in. at the tip.

Very little distortion occurred, particularly cross-sectional distortion. A set procedure was adopted in an endeavour to minimise distortion, based on the following reasoning. If ultimate strength be plotted against age-hardening time, the shape of the curve was roughly as shown. From this it could be seen that no material change in properties (in fact even a slight reduction) could occur during the first half hour or so. Now the essential parameter influencing the release of thermal stress, after machining, during the age-hardening process was temperature and it was considered that 90 per cent. of any distortion likely to occur would take place within half an hour soaking time at the age-hardening temperature. Thus it was decided to rough machine in the solution treated condition, take the plank up to the age-hardening temperature for half an hour, during which most of the thermal stresses should have been relieved with little change in mechanical properties, final machine and then final age-harden.

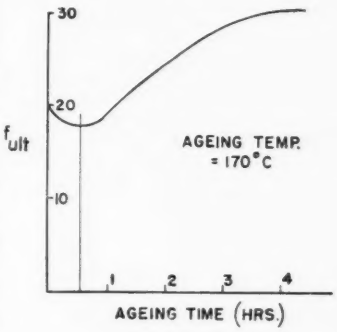

FIGURE 2.

MR. GIDDINGS (Bristol Aeroplane Co.): He disagreed with the applicability of the various types of construction as shown on Mr. Keen's Fig. 5 and felt that the case had not really been made for integral construction. He agreed with the ultimate use of multi-web construction for the very high loadings, but considered that both the stringer-sheet and multi-web designs could either be riveted or of integral machined fabrication with little change in technical efficiency. He pointed out that the problem was not only one of strength, but also of stiffness, and the most economical distribution of skin thickness might be one of constant thickness from root to tip. In the circumstances he felt that the only possible weight saving would be that caused by reduction in the number of rivets used, which he felt was not sufficient

*"The Design of Compression Structures for Minimum Weight." D. J. Farrar. *Journal of the Royal Aeronautical Society*. November 1949.

compensation for using integral construction rather than stringer-sheet. He thought, therefore, that the trend was from stringer sheet direct to multi-cells.

Regarding Mr. Legg's contribution he considered that the type of section used must be inefficient due to the possibility of torsional instability of the upstanding web.

MR. KEEN: In replying to the criticism of his Fig. 5, he pointed out that for the higher loads per inch he had introduced the supposition that the stringers would be limited to 2 in. deep, which would probably account for the discrepancies between Mr. Giddings's views and his own.

He thought that the use of integral construction should not only be considered from structural aspects but also from a production point of view. His own company, for example, disliked intensely the enormous amount of riveting required for skin stringer sheet and actually preferred integral construction on the grounds of reduced assembly time and number of parts involved.

The point about the torsional stiffness and bending strength requirements producing a constant skin thickness from root to tip for optimum weight conditions was of considerable interest, but was based on the present method of presentation of stiffness requirements. A change in this method, which was a distinct possibility, might produce a different answer regarding skin thickness.

MR. HOWARD (Ministry of Supply): Assuming integral construction was coming in, as he had no doubt it was, would it be introduced at the prototype or production stage? Listening to the discussion so far, he formed the impression that the emphasis in research work might be more on stress distributions in thick sheets and, possibly, their fatigue than on the investigation of the various instabilities in compression.

MR. KEEN: In his opinion integral construction would have to be introduced at the prototype stage as the change-over would probably result in considerable expense and delay and in any case the structural form would be very different.

Regarding further research work, there would still be large fuselages requiring thin skin integral type of construction (possibly extrusions) and in his opinion there was a great need for more investigation into stability in compression. In addition there was a need for considerable expansion of research in all directions, including heat treatment and distortion.

MR. LEGG: Replying to Mr. Giddings's point on torsional instability. Initially it had been considered that torsional instability would be a dominating feature in design. Current available theoretical work, however, indicated that the only torsional instability likely to arise would be from secondary warping due to flexure. Panels of representative flexural lengths and good structural efficiency had been tested and high compressive stresses had been achieved. The problem of secondary warping due to flexure was still being investigated.

Regarding the advantage of integral construction from a weight point of view it was quite certain that for some time to come not all wings would have a constant skin thickness and, where bending strength was the criterion, the sharp rise in the bending moment curve at the root caused a considerable loss in weight due to the maximum bending material requirements when stringer-sheet was employed. The spar box project referred to previously showed an approximate weight saving of 25 per cent. over the original skin stringer construction.

MR. R. BOORMAN (Short Brothers and Harland Ltd.), *contributed*: In support of the views expressed by the lecturer he thought that one way of combating the structural disadvantage that thin wings were forcing upon them was to redistribute the structural material nearer and nearer to the skin profile; if this could be done and at the same time a saving could be effected in the number of parts which constituted the main strength-carrying member of a wing not only would a more efficient structure be provided but also, one with definite production advantages in time and cost.

Some would shy at the large initial cost and the complexity of machine tools necessary to produce integral structures, but he expected that some had shied at the first spar millers—the spars from which were after all, nothing more or less than the beginning of integral construction. The present trend was merely taking this type of construction one stage nearer to what might be its ultimate conclusion—a one-piece wing free from all joints, rivets, bolts and other stress raisers and fatigue promoters—free of redundant material due to wasteful overlaps of skin and stiffeners at joints.

The lecturer had not spotlighted that particular aspect but he felt that improvement in fatigue properties would also " weigh in " on the side of integral construction.

Any diminution in number of parts was accompanied by a reduction in the number of holes necessary to join those parts, and the less the number of such stress raisers, the happier and longer the life of the structure.

It was true that the actual machining costs would be higher than those to which they were accustomed with the present built-up structures, but it was equally true that there would be a saving in production control, stocking, inspection, transportation of bits and pieces, tool planning, assembly labour and assembly tooling, and " paper-work "—all of which were probably proportional to the number of parts to be put together.

Mr. Legg had already outlined briefly the spar box being constructed by his company, but to illustrate the point made in the preceding paragraph, he would say that the upper spar plank alone replaced no less than 188 pieces of sheet or folded section and some 6,000-7,000 bolts, nuts and rivets.

It was also estimated that the saving in overall costs in production, including the increased machining cost, was of the order of 30 per cent.

The necessity of machining the upper and lower planks from solid slabs increased the cost of material to nearly double, but even at that figure the total material costs were only 13 per cent. of the overall cost of the specimen.

TECHNICAL NOTES

The increasing flood of technical information makes it difficult to publish even abstracts of such information and it has become still more difficult to publish technical items of information which do not reach the status of full-length papers.

Under this general heading of TECHNICAL NOTES it is hoped that members of the Society, and indeed all those who read THE JOURNAL, *will help in contributing just those very items of useful information which normally might not see the light of the printed page.*

There will be no set form for these Notes. They may be illustrated or not. They can be short papers of a thousand or two thousand words; or abstracts and comments upon unpublished papers or upon published papers with a limited circulation; or notes of some interim results of research in hand and notes for further research; or in the form of a letter raising technical points or asking questions; or letters commenting on Notes published.

These pages are intended to be a technical forum where members can argue and discuss; where they can inform and be informed; and pass on their own experience, or air their technical difficulties. Contributions will be eligible for THE JOURNAL *Premium Awards and they will be published as soon as possible after they are received.*

No member need ever say that his particular subject does not get discussed in the pages of THE JOURNAL. *He can always raise it himself, for these pages are his pages.*

This month we publish a comment on one of the Notes in the April Journal, a brief and possibly controversial note on inspection and measurements for production, and a note on a method of obtaining a fuselage air-load distribution for a swept-back wing aircraft.—THE EDITOR.

Compressor Design

by

R. G. TAYLOR

(Bedford College, University of London)

IN Mr. J. M. Stephenson's Technical Note, "The Elimination of Wall Effects in Axial-Flow Compressor Stages," in the April 1953 issue of the JOURNAL, the author suggests that the blade rows of an axial flow compressor are so closely spaced as to ensure that the axial velocity profile is unchanged across the rows. Whether this statement is correct or not such an assumption regarding the axial velocity profile is a basic design condition and when made it will not leave any flexibility in the choice of the function $f(r)$. If the axial velocity profile is the same before and after the rotor row, then equation (9) implies

$$w_0\left(\frac{w_0}{r}+\frac{\partial w_0}{\partial r}\right)=w_3\left(\frac{w_3}{r}+\frac{\partial w_3}{\partial r}\right),$$

and this completely determines $f(r)$, giving $f(r)$ proportional to r. In fact if the total head before the rotor is constant radially (equation (8)), the work done across the row is constant radially (Section 1), there is no radial velocity (Section 2) and $v_{a0}=v_{a3}$ (Section 2), then the air flow is the familiar free vortex.

To get away from this type of flow, one of these design conditions must be relaxed. It is perhaps worth commenting, although this is not the place to develop the idea, that the assumption of no radial variation in energy has come to be regarded as an almost inviolable rule of compressor design, but there does not appear to be any substantial reason why such a restriction should be applied to every stage. If this restriction were relaxed, even for only one stage in a compressor, it would result in a valuable increase in freedom of design. The writer would be very glad indeed to know if any information about the performance of such a stage or number of stages is available.

Received 23rd May 1953

Dimensions Required for Inspection

by

G. A. G. SAYWELL

(Formerly Chief Inspector, Aircraft Division, with a Sub-contractor)

THE following problem constitutes a minor delay in ultimate production, yet considerable time and effort and manpower are duplicated at present in finding a practical solution to what is really a repetition of Drawing Office calculations.

The problem is that drawings calling up various machined fittings often lack any reference to "practical" measurements by which the operator, the toolmaker and the Inspection Department may "set-out" and check the work concerned.

Measurements, often to fine limits, given on drawings, can rarely be used in the "direct" sense for checking purposes, *i.e.* by means of surface table and height gauge, and so on.

The normal practice is for the various departments to evolve a set of "practical" calculations from which direct measurements may be made. The accuracy of these calculations is often the subject of much controversy between departments with consequent time lag. Particularly does this apply on work involving sub-contractors where Drawing Office facilities are not readily available. Close liaison between departments is required to ensure that all work to the same degree of accuracy.

As most of the essential calculations will have been made by the Drawing Office staff at some stage, is it not reasonable and practicable for sets of "direct" measurements to be included with the normal drawing particulars?

A typical example is shown in Figs. 1 and 2. It is a simple one, but it illustrates the sort of calculations which are made repeatedly by various departments.

Received 21st April 1953

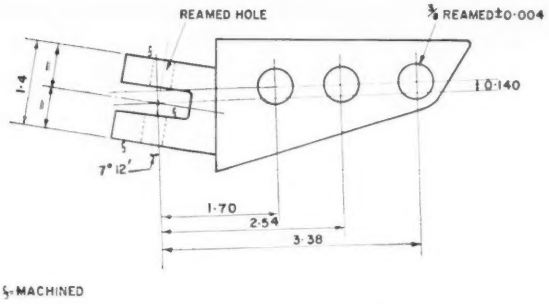

FIGURE 1. Drawing details given.

In Fig. 2, the height-gauge reading to the top of the end hole

$$= a + b + c + d + e$$
$$= \text{height of parallel bar} + 0{\cdot}7 + 3{\cdot}38 \sin 7^\circ 12' + $$
$$+ 0{\cdot}140 \cos 7^\circ 12' + 0{\cdot}375/2$$
$$= 1{\cdot}0 + 0{\cdot}7 + 0{\cdot}4235 + 0{\cdot}1388 + 0{\cdot}1875 = 2{\cdot}4498.$$

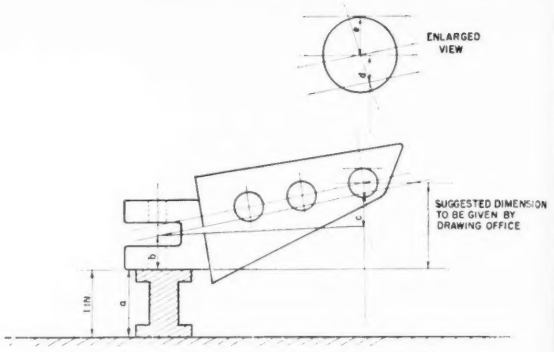

FIGURE 2. Drawing details required.

Often the machined face, shown resting on the parallel bar is a compound angle, which makes the calculations more complicated.

The "practical" dimension required is also shown in Fig. 2, and it would be interesting to have the comments of readers on this, and on the problem in general.

Fuselage Lift Distribution for a Swept-Back Wing Aircraft

by

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(Vickers-Armstrongs, Supermarine)

THIS Note suggests a method, suitable for stressing purposes, of obtaining a fuselage air-load distribution, for a swept-back wing aircraft. The method is basically that given in Ref. 1. Mach number effect on the aerodynamic centre position is discussed and an approximate method of allowing for this is suggested. A numerical example is given.

1. INTRODUCTION

A wing aerofoil of symmetrical section and a body which is symmetrical in side view and plan view (not necessarily a body of revolution) is considered throughout. Hence the term "aerodynamic centre" refers also to the centre of pressure.

The loading on a fuselage is suitably divided into three parts:—

(*i*) and (*ii*). Loading ahead of and behind the wing which is dependent on upwash and downwash, respectively, and body shape.

(*iii*). Fuselage lift in the way of the wing, *i.e.* that portion of the wing shielded by the body. Here the conception is that there is a quantity of wing lift distributed according to the aerofoil chordwise pressure distribution for that portion of the undisturbed swept-back wing. This particular "piece of lift" is decreased by the shielding, and the centre of this loss is approximately at the C_L quarter-chord point[1].

Received 15th April 1953

NOTATION

A	aspect ratio
a_1	lift-curve slope of the undisturbed wing $= dC_L/d\alpha$
$a_{1(100)}$, a_{1WB}	used and defined in Section 2.2
b_B	body width (a function of x) (in.)
c_B	mean chord of the wing in the way of the body (defined in Fig. 1)
$c_{C.L.}$	chord of the wing at the aircraft centre line (*see* Fig. 1)
C_{LB}	body lift coefficient
k	defined by $ka_1 = dC_{LB}/d\alpha$
C_{M0}	pitching moment coefficient at zero lift
L	loading ordinate in lb. for a one-inch strip in the x-direction
P	relative pressure on aerofoil $= p/q$
P_a	value of P for additional part of pressure distribution, when the additional section lift coefficient is 1·0
p	local aerofoil pressure (lb./in.^{2})
q	dynamic pressure (lb./in.^{2})
s	wing semi-span (perpendicular to x-axis)
x	co-ordinate measured along the fuselage longitudinal axis
x_1	distance ahead of the C_L quarter-chord point parallel to the x-axis

α wing incidence

β angle of downwash to body no-lift line (function of x)

ξ_1 x_1/s

Λ angle of sweepback of quarter-chord line

S_w gross wing area

S_B wing area in way of body (*i.e.* area included between wing leading edge and trailing edge extended to centre line of aircraft)

2. CALCULATION OF THE ADDITIONAL LOADING

The loading distribution from (*i*), (*ii*) and (*iii*), is referred to as the "additional loading," being associated with the aircraft attitude, and is calculated in Sections 2-4 for the example shown in Fig. 1. The constant part of the loading is dealt with in Section 5.

2.1. *Calculation of downwash and additional lift*

The downwash $\beta = \dfrac{d\beta}{d\alpha}\alpha + \text{constant}$

where[1]

$$\frac{d\beta}{d\alpha} = 1 - \frac{a_1}{2\pi A}\left\{1 - \frac{\sqrt{\{(\xi_1 + \tan\Lambda)^2 + 1\}} - \left(\frac{\xi_1}{|\xi_1|}\right)\tan\Lambda}{\xi_1}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The following data are required:—

Wing lift-curve slope,	$a_1 = 2{\cdot}5$
Aspect ratio,	$A = 2{\cdot}1$
Semi-span,	$s = 150$ in.
Quarter-chord sweepback angle,	$\Lambda = 45°$.

Using this, $a_1/(2\pi A) = 0{\cdot}19$, and $d\beta/d\alpha$ is calculated from equation (1). The results are given in Table I, but the full tabulation is not shown.

The additional lift can be expressed as

$$\frac{1}{q}\left(\frac{dL}{dx}\right) = \left(\frac{\pi}{2}\right)\frac{d}{dx}(\beta b_B^2) \quad . \quad . \quad . \quad (2)$$

TABLE I

CALCULATION OF $d\beta/d\alpha$
(REGION AHEAD OF WING LEADING EDGE AND BEHIND WING TRAILING EDGE

x: in.	$d\beta/d\alpha$
150	1·045
135	1·052
120	1·062
90	1·091
60	1·152
30	1·344
10	2·130
−150	0·430
−180	0·489
−210	0·527
−240	0·553
−270	0·569
−300	0·580
−330	0·589
−350	0·593

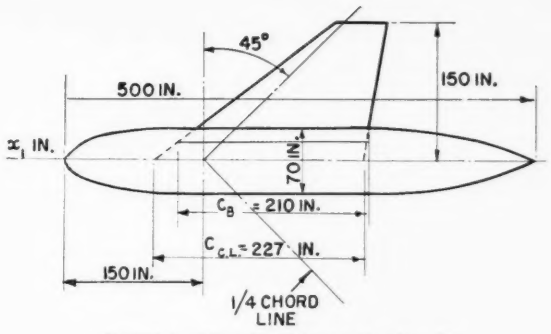

FIGURE 1. Plan view of example aircraft.

Differentiating with respect to α this becomes

$$\frac{1}{q}\frac{d}{d\alpha}\left(\frac{dL}{dx}\right) = \frac{\pi}{2}\frac{d}{dx}\left(\frac{d\beta}{d\alpha}b_B^2\right), \quad . \quad (3)$$

where L is the loading in pounds on a one-inch strip in the direction of x and b_B inches can be scaled from a plan view of the fuselage, assumed constant in the way of the wing.

Values of the function of equation (3) are tabulated in Table II, the results being shown in column (6).

TABLE II

CALCULATION OF THE ADDITIONAL LIFT
(REGION AHEAD OF WING LEADING EDGE AND BEHIND WING TRAILING EDGE)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
x: in.	b_B: in.	$\frac{\pi}{2}b_B^2\frac{d\beta}{d\alpha}$	Difference in Col. (3)	dx	$\frac{\text{Col. (4)}}{dx}$	Mean x_1
150	0	0				
			1587	15	105·8	142·5
135	31	1587				
			1356	15	90·4	127 5
120	42	2943				
			2625	30	87·5	105
90	57	5568				
			2314	30	77·1	75
60	66	7882				
			2169	30	72·3	45
30	69	10051				
			6344	20	317	20
10	70	16395				
−150	70	3310				
			454	30	15·1	−165
−180	70	3764				
			54	30	2·1	−195
−210	68	3828				
			−596	30	−19·9	−225
−240	61	3232				
			−907	30	−30·2	−255
−270	51	2325				
			−939	30	−31·3	−285
−300	39	1386				
			−1016	30	−37 2	−315
−330	20	370				
			−370	20	−18·5	−340
−350	0	0				

TABLE III

CALCULATION OF REDUCED LIFT SLOPE OF WING OVER THAT PORTION SHIELDED BY BODY

(1)	(2)	(3)	(4)
C_L	$\frac{\text{Fuselage Lift}}{\text{Total Lift}}$	$C_{LB}=C_L\times$ Col. (2)	$\alpha=C_L/2{\cdot}5$ (*radians*)
0·25	0·190*	0·0475	0·1
0·5	0·216	0·108	0·2
0·75	0·223	0·167	0·3
1·0	0·225	0·225	0·4

*From relevant section of Air Publications Manual or alternative source.

2.2. *Calculation of lift distribution over that portion of the body covering the wing*

The lift distribution over the body in way of the wing can be written

$$\frac{1}{q}\left(\frac{dL}{dx}\right)=C_{LB}P_ab_B=ka_1\alpha P_ab_B=\frac{S_w}{S_B}ka_1\,\alpha\,P_ab_B \quad (4)$$

Differentiating with respect to α gives

$$\frac{1}{q}\frac{d}{d\alpha}\left(\frac{dL}{dx}\right)=\frac{S_w}{S_B}ka_1P_ab_B \quad . \quad . \quad (5)$$

The next step is to calculate ka_1 as in Table III and Fig. 2.

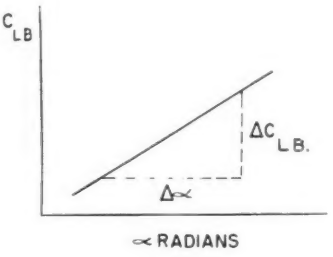

FIGURE 2. Fuselage lift coefficient curve.

Plotting C_{LB} against α as in Fig. 2 gives

$$\frac{dC_{LB}}{d\alpha}=ka_1=0{\cdot}593.$$

The lift distribution given by equation (5) is now calculated using, for the example,

$$S_w=298 \text{ ft.}^2$$
$$S_B=102 \text{ ft.}^2$$

Now $$a_{1wB}=ka_1\frac{S_w}{S_B}=1{\cdot}73$$

and $a_{1(100)}$ is taken equal to $0{\cdot}305\ a_1\frac{S_w}{S_B}=2{\cdot}23$,

where 0·305 is the proportion of lift over the fuselage for 100 per cent. efficiency, obtainable from the spanwise Load Grading Curve. (If not known, this may be obtained from Jones's simplified method[2].)

With $c_B=210$ in. and $c_{C.L.}=227$ in. (*see* Fig. 1), the calculation proceeds as shown in Table IV. Col. (2) is obtained from the aerofoil chordwise pressure distribution.

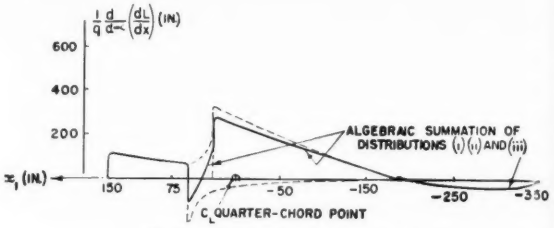

FIGURE 3. Additional lift curve.

3. THE "FUSELAGE ADDITIONAL LIFT CURVE"

A graph of the "fuselage additional lift" is plotted as follows.

Set out a horizontal axis equal to the length of the fuselage in inches to some suitable scale, on it mark the extent of c_B and $c_{C.L.}$ and the position of the $c_{C.L.}$ quarter-chord point. First plot the 100 per cent. lift distribution along the chord c_B (*i.e.* Col. (3) of Table IV) and next plot the loss of lift (whose centre of pressure is at the $c_{C.L.}$ quarter-chord point) along the chord $c_{C.L.}$ (*i.e.* Col. (7) of Table IV).

The algebraic summation of these two distributions will then give the true* or resultant curve in the way of wing (*i.e.* distribution (*iii*)). The last step is to plot the distributions (*i*) and (*ii*) ahead of and behind the wing (*i.e.* Col. (6) of Table II) giving Fig. 3.

On the argument that the fuselage never stalls before the wing, the fuselage lift is always linear with α; therefore the lift curve for any aircraft attitude can be obtained directly merely by multiplying the ordinates of Fig. 3 by the product of q (lb./in.^{2}) and α (radians) (corresponding to the aircraft's equivalent air speed and attitude).

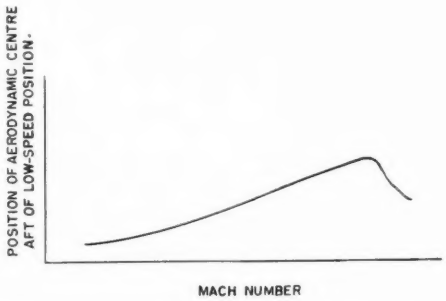

FIGURE 4. Shift of fuselage aerodynamic centre with Mach number.

In practice, however, it is preferable that the area under the curve of Fig. 3 represents a fuselage lift of

*Strictly the 100 per cent distribution should extend from the leading edge of the centre-line chord to the trailing edge of the chord at the fuselage side; if the consequent small discontinuity involved proves embarrassing the appropriate adjustment can be made.

TABLE IV

CALCULATION OF LIFT DISTRIBUTION

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Percentage Station on c_B	P_a *c.p. at* $0{\cdot}34\,c_B$	$a_{1(100)}\,P_a b_B$	*Station from Leading Edge: in*	*Inches aft of C_L quarter-chord point*	P_a { *with c.p. at quarter-chord point*	$(210/227)$ $(a_{1(100)}-a_{1WB})\times b_B P_a$	*Inches aft of C_L quarter-chord point*
1·25	2·03	316	2·6	−25·1	6·08	−197	−53·8
2·5	2·06	321	5·2	−22·5	4·4	−143	−51·0
5	2.04	319	10·5	−17·2	3·24	−105	−45·4
7·5	1·96	306	15·7	−12·0	2·67	− 86	−39·7
10	1·87	291	21·0	−6·7	2·30	− 75	−34·0
15	1·72	268	31·5	3·8	1·81	− 59	−22·7
20	1·60	250	42·0	14·2	1·50	− 49	−11·3
30	1·36	212	62·9	35·1	1·12	− 36	11·3
40	1·20	187	83·9	56·1	0·87	− 28	34·0
50	1·00	156	104·9	77·1	0·67	− 22	56·7
60	0·80	125	125·9	98·1	0·50	− 16	79·4
70	0·59	92	146·8	119·1	0·36	− 12	102·1
80	0·39	61	167·8	140·0	0·22	− 7	124·7
90	0·18	28	188·8	161·0	0·09	− 3	147·5
95	0·09	14	199·3	171·5	0·02	− 0·6	158·8
						Loss of Lift	

say, 10,000 lb. The reason for this is that when carrying out balance calculations the fuselage lift will be appropriate to the value of lift coefficient read off the fuselage lift curve (with allowance for Mach number effects). Conversely when stressing the fuselage, the fuselage lift will be taken from the particular balance case considered. Hence it is easier to factor up Fig. 3 by an amount such as "fuselage lift from balance case in pounds" divided by 10,000 lb.

4. MACH NUMBER EFFECTS

The first requirement here is a graph showing shift of fuselage aerodynamic centre with Mach number similar to Fig. 4. As the major part of the lift acts along the chord c_B and the shift of the aerodynamic centre is relatively small it is justified to apply the correction in this region alone (*i.e.* along c_B). This graph is really a shift of the aerodynamic centre of the wing in the way of the body.

A sinusoidal distribution, as shown in Fig. 5 is suggested for a high-speed correction to Fig. 3. This may be considered rather crude but may be justified because the amount of lift involved in relation to the total lift is small.

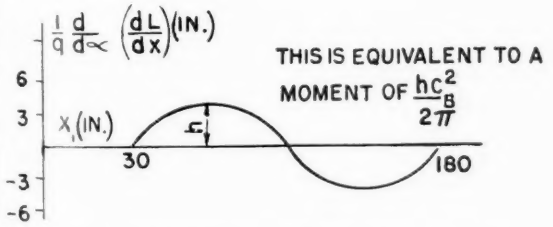

FIGURE 5. High-speed correction for a one inch shift of aerodynamic centre.

To obtain the effect of a shift of aerodynamic centre of h inches aft, subtract h times the distribution of Fig. 5 from the low-speed curve of Fig. 3. Once more the loading scale must be adjusted if Fig. 3 is given in terms of a total lift of 10,000 lb.

5. THE BASIC OR C_{M_0} DISTRIBUTION

More often than not the no-lift line of the fuselage and the wing do not coincide; in this case the constant part (or basic loading) must be considered at the condition $\alpha = 0$.

Here the air is flowing parallel to the wing chord and the wing is in a state of no lift; the only lift, therefore, is emanating from the "fuselage setting" and/or from protuberances such as the cockpit hood. The condition most nearly approaching this would be in the dive*. Tunnel tests will give the maximum value of C_{M_0} likely to be met in practice, probably of the order of $-0{\cdot}01$. As this would have to be used in balance calculations, the corresponding value of the moment at zero lift can be obtained from the particular balance case considered. The graph of basic loading can be constructed using that part of the distribution in Fig. 3 which extends ahead of and behind the wing (taking account of sign).

REFERENCES

1. SCHLICHTING, H. (1952). Calculation of the Influence of a Body on the Position of the Aerodynamic Centre of Aircraft with Swept-back Wings. R. & M. 2582, 1952.

2. STANTON JONES, R. (1950). An Empirical Method for Rapidly Determining the Loading Distributions on Swept-back Wings. College of Aeronautics Report No. 32, January 1950.

*This is not strictly true as the Dive Case would be taken as with the total net aircraft lift equal to zero.

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Reviews

STRENGTH AND TESTING OF MATERIALS. *Parts I & II. Department of Scientific and Industrial Research. Selected Government Research Reports, Vol. 6. 260 pp. and 244 pp. Diagrams. H.M.S.O. 1952. 55s. each part.*

Doubtless it is inevitable that a published book should be given some kind of descriptive title. Whistler could paint a picture of Chelsea reach and call it a Symphony in Blue and Gold; a poetaster can publish his slim volume and dub it Mauve Muslin though it deals mostly with codfish and absinthe; but H.M. Government must eschew such evasions. This being so it is possible that the title chosen for these two excellent volumes may be as good as any other, for their contents cover so many different topics relating to materials that the only really valid title would be a schedule of the several papers and reports that have been bound up together. For a publication that contains contributions on such diverse topics as The Theory of Autofrettage, The Theory of the Extrusion of Metals, The Detection of Peel on White Heart Malleable Iron Track Links, The Distribution of Stress in a Decelerating Elastic Sphere, together with Middleton's investigation of the Distribution of Stress in the Vicinity of a Spherical Inclusion, it is virtually impossible to discover a titular, highest common factor. It seems possible therefore that these volumes may not contain exactly the matter which a purchaser might expect to find.

A rather similar difficulty arises in providing an adequate appreciation of this publication in any reasonable number of words. The topics examined are too diverse, and each paper is worthy of a separate consideration. The thirty reports are not all the work of members of either A.R.D. or R.A.E.; some were produced under extra-mural contracts, notably that on Notch Brittleness and Ductile Fracture by Orowan, Nye and Cairns, of the Cavendish Laboratory. Volume 1—the section acknowledgely theoretical—contains, in addition to some of those already mentioned, theoretical papers on the indentation of ductile materials, on the theory of combined plastic and elastic deformation, on the compression of a body between rough plates, on the two-dimensional theory of plastic yielding of deeply-notched bars in tension, and on general methods of solution of the axially symmetric problem in plasticity. The report by Orowan, Nye and Cairns is the longest. The work that it covers was brilliantly executed, and the conclusions reached have exercised a powerful influence on all the subsequent thinking about the fracture of metals when notched, as all are—in some degree—in practical engineering. The second volume, entitled Testing Methods and Test Results, contains seventeen reports. Three are concerned with the statistical examination of test factors and strength variation; six deal with various aspects of steel-welding, being particularly concerned with the results given in actual tests; while three are more or less theoretical papers on problems relating to elasticity. One, longish, report gives a large number of results obtained in bearing tests on tubes, bars and sheets of different aircraft materials.

Little that is not laudatory can be written of this two-volume publication. It is likely to be highly valuable, not so much to those engaged in the practical testing of metals as to those wishing to understand the nature of the deformations, both elastic and plastic, that are suffered by materials of construction. The theoretical papers, as fully mathematical as would be expected, and adequately illustrated, are generally pointed and succinct. The more workaday reports on tests obtained in practice are perhaps not so attractive. Doubtless it was appropriate and necessary, when the reports were originally made, to record all the test-results obtained, but it seems rather unnecessary at this date to reissue them in full detail. If the reports had been condensed and summarised, with the conclusions clearly emphasised, their value would not have been diminished. It is surely true that all the valuable knowledge which emerged from these investigations has already been absorbed into practice and that the reports achieved their purpose when originally issued. On the other hand, the theoretical papers are of abiding interest and value. By their very nature they must be reissued in full completeness.—LESLIE AITCHISON.

INSTRUMENT ENGINEERING, VOL. I. *Methods for Describing the Situations of Instrument Engineering. Draper, McKay and Lees. McGraw-Hill 1952. 269 pp. Index. 51s.*

This book, which is pleasantly printed by offset reproduction, forms part of a major work in three volumes. The keynote of the whole work is that of generalisation, both physically, where the symbolism is applicable to all scientific and engineering fields, and mathematically. The well accepted practice of teaching from the particular to the general, from the concrete example to the abstract concept, is not carried out, this being contrary to the main theme. The three parts of "Instrument Engineering," which divides into three volumes, can be classed broadly as:—Formulation of physical problems arising in automatic control systems (Vol. 1). Solution of the Formulæ (Vol. 2). Application to particular systems (Vol. 3).

In Volume one the authors have developed an elaborate system of notation to enable a general formulation of the performance of feedback systems to be accomplished. The notation takes the form, essentially, of initials and abbreviations; Q for quantity, $Q_{(\text{emprsn})}$ for comparison quantity. $[PF]$ implies the performance function of an element or of a system which can be identified by subscripts, for example $[PF]_{(\text{osegs})}$ is the performance function of the output signal correction generating systems. This notation places a great strain on the memory of the reader and has the additional disadvantage of making the most simple equation apparently complicated. If this notation must be used a complete list of the symbols adopted would be a help; it is impossible to delve into any section without first having read the previous part of the whole book.

Stability is described in a chapter on Dynamic Performance by classifying possible output motions as transiently stable or unstable and possessing hunting stability or instability. Other chapters worthy of mention are one on the gyroscope, one on dimensional analysis and one on statistical methods; this latter chapter, written by H. B. Brainherd, is a useful addition.

It is doubtful whether this book will be of use to the student or the practising engineer. Its generalised method of attack makes it essentially of interest to specialists in the field. Apart from the generalisation and the notation there is little that is new or original.—J. C. WEST.

WELDING PRACTICE. *Vols. 2 and 3. Edited by E. Fuchs and H. Bradley. Butterworths Scientific Publications. London 1952. Vol. 2, Welding of Ferrous Metals. 198 pp. Vol. 3, Welding of Non-Ferrous Metals. 183 pp. Illustrated. Each 22s. 6d. net.*

A review of the first volume of this trilogy appeared in the July 1952 issue of the JOURNAL, and now both the second and third are available. As so much time has elapsed since the appearance of the first volume, a word as to the origin of the series is appropriate. It is not the work of a single author, but has been produced by some dozen members of the Welding Panel of Imperial Chemical Industries Ltd.

In his review of the first volume, the present writer said, *inter alia*, that with so many highly qualified contributors, one felt some disappointment. It would have been unfair to have suggested that there was some analogy to the mountains having been in labour and having brought forth a mouse. On the other hand, it was considered legitimate to say that had the Editors been sterner, the book would have been better.

Volume 2, now reviewed, deals with the welding of ferrous metals. Of this, the writer has little but warm praise. It contains seven chapters. Five deal lucidly and comprehensively both with welding processes and with their application to mild steel, high-tensile low-alloy steels, chromium-nickel austenitic steels and to cast iron. Of the remaining two chapters, one relates to weld details for pressure vessels and the other deals with hard surfacing by the use of welding techniques. But two criticisms of this excellent volume can be made. The post-welding treatment of parts likely to be subjected to cyclic stresses is too condensed, a criticism which can also be applied to that relating to the important subject of pressure welding.

The third and final volume of the series is almost wholly devoted to the welding of non-ferrous metals. It contains seven chapters, of which six actually deal with welding, while the seventh gives an exposé on brazing techniques. The metals and their alloys whose joining is considered, are copper, brasses and bronzes, nickel and nickel-rich alloys, silver, aluminium and aluminium alloys and lead—the aeronautical engineer will be as quick to note, as he is to regret, the omission of magnesium and its alloys.

This volume is definitely good, although possibly not quite up to the exceedingly high standard of that on the welding of the ferrous metals. One notes with satisfaction the emphasis on such metallurgical factors as the presence of oxygen in tough pitch copper, with the consequent danger to it from contact with a reducing flame—the singular susceptibility of nickel and of many nickel alloys to sulphur contamination, possibly giving rise to serious embrittlement—the equally disastrous results brought about by absorption of oxygen during the fusion welding of silver and its alloys, etc.

Finally, to give an overall evaluation of this trilogy—it is a workmanlike but not a perfect piece of work. In the next edition, and its merits are such as to make sure one will be required, the engineers among those responsible for its production must be much more assertive. It is most desirable that they say much more about the mechanical properties of the various types of welds both under static and, what is more important, under *dynamic* conditions. The too scant treatments of techniques of such tremendous potentiality as pressure welding and also of the self-adjusting argon arc process must be appropriately expanded, and what to the aeronautical engineer is an almost unpardonable omission, the absence of any reference to the welding of magnesium alloys, must be rectified. These are faults and not insignificant ones, yet there is so much that is excellent in these three volumes that they are strongly commended, both to designers and to production engineers.—P. L. TEED.

PRINCIPLES OF AERODYNAMICS. *Daniel O. Dommasch. 389 pp. Diagrams. Pitman, London 1953. 45s. net.*

Professor Dommasch's book, which appears to be intended mainly for students, is concerned largely with the theory of inviscid incompressible and compressible flow, discussion of the influence of viscosity being confined to the final chapter. The author devotes approximately equal space to incompressible and compressible flow problems and makes a particular point of the detailed explanation of the steps in the derivation of the equations. This generally laudable practice is open, in the present instance, to two criticisms; firstly the methods employed in deriving some of the more important results, particularly in the chapters on compressibility, are not the most concise and direct, and secondly in some places space is wasted by unnecessary repetition.

Following a first chapter on general principles the next five chapters, including one on complex number and vector theory, are devoted to incompressible problems mostly of a two-dimensional nature, although a short account of the Lanchester-Prandtl wing theory is given in Chapter 6. The treatment follows largely standard lines, although it is somewhat unusual to find the velocity potential defined before circulation and vorticity and this inversion of the conventional order would seem to lead to a number of difficulties without, so far as the reviewer can see, any compensating advantages. It is a pity that no attempt has been made in this section, or later, to illustrate differences between theory and experiment and that the few remarks on this subject are, in places, misleading.

The compressible flow section covers a wide field, including both two- and three-dimensional linearised theory for supersonic flow, although it is perhaps understandable that the detailed derivation of results, practised elsewhere in the book, has been omitted in the latter case. Ackeret's theory is clearly described and the limitations to its application are fully discussed; the treatment of the Prandtl-Meyer expansion and the inclined shock wave are less satisfactory and may well lead the uninitiated to view these not too difficult matters with apprehension.

The final chapter dealing with viscosity effects begins with the derivation of the Naviar-Stokes equations and proceeds to discuss laminar and turbulent boundary layers. The description of transition and separation is lacking in accuracy and does not give a clear picture of the changes in the flow near the surface of an aerofoil with increase of Reynolds number.

The book cannot be recommended unreservedly to students new to the subject, but should be of considerable value to those already partly acquainted with the subject and requiring a more detailed knowledge of the matter covered. A number of important topics, such as aerofoil cascades, hodograph methods for subsonic flow and the influence of the restricted dimensions of experimental air streams, are not mentioned.

The book is well arranged and only a few minor misprints have been noticed. Some of the diagrams are too small and consequently lack clarity, while the tables for compressible flow included at the end of the book have only limited value as the intervals used for Mach number are too large.—L. G. WHITEHEAD.

STATISTICAL QUALITY CONTROL. *Eugene L. Grant. McGraw-Hill 1952. 558 pp. 94 illustrations. Index. 55s. 6d.*

This is the second edition of Professor Grant's well-known textbook on Quality Control.

When the first edition appeared in 1945/47, there was very little published in this field, although a lot of work had been done during the Second World War in both Britain and the United States. Since then many books have appeared, some of them deeper and more learned, but none of them more comprehensive than Grant's "Statistical Quality Control."

The second edition differs in detail from the first chiefly in the addition and amplification of sections dealing with multiple and sequential sampling (when attributes are the basis)—a technique released from the bonds of secrecy after 1946. It differs also by the addition of a new chapter on "Acceptance Sampling by Variables," a technique which, when properly understood, may permit smaller samples and frequently points more quickly and clearly to technical troubles and to their solution. This is an often overlooked virtue of quality control in general and of sampling by variables in particular.

As might be expected from Professor Grant, who occupies the Chair of Economics of Engineering at Stanford University, there is a thorough discussion of the cost aspects and the organisation and industrial applications of quality control. One feels throughout the book that it is not the mathematical elegance of the techniques but the great potential saving of effort and in manpower that really inspires him.

Criticised from the point of view of a pure statistician, there are some defects in the treatment of the subject but from the point of view of the student and the production engineer wishing to get to grips with Quality Control as a *tool*, this is a most useful book. It also goes right through to the bitter end and gives many worked examples and some case histories.

The spread of Quality Control in this country is at present hindered (probably inhibited is a better word) by the complete inability of much of management to understand the theory behind it. The reaction is predictable—and then one cannot expect the "100 per cent. inspector" to destroy deliberately much of the basis for his existence when management will not force the issue on economic grounds.—J. V. CONNOLLY.

WRITING THE TECHNICAL REPORT. *J. Raleigh Nelson. McGraw-Hill 1952. 356 pp. Third edition. 38s. 6d. net.*

That this excellent book has reached a third edition since its first appearance in 1947 shows that its value is appreciated. It meets a need that is being increasingly recognised by industrialists on both sides of the Atlantic, the need to improve methods of communicating technical information from one person to another. Nearly all industrial effort is co-operative nowadays, and becomes increasingly so with the increasing complexity of modern civilisation, and so effective communication within any organisation, as well as between organisations, has become vastly more important than it was but a generation ago. It is no exaggeration to say that talk and paper are now among the more important means of production. A report that is hard to understand may waste as many precious man-hours in an industry as may a machine that takes excessive time to manipulate or adjust.

Although this book is specifically concerned with one means of communication only, the technical report, much of the advice is also applicable to those who may have to prepare maintenance or erection instructions, publicity material, minutes and memoranda of meetings, written and verbal instructions, papers to learned societies. Much the same basic technique is required for all such purposes and the effectiveness of our industrial machinery and organisation depends on this technique to a much greater extent than is often recognised.

It is often said that the appropriate place for teaching what a writer of good reports needs to know is the school. Those who think so gravely fail to appreciate the problem. It is not one of English. A master of grammar, syntax and style, may fail badly in industry when he is asked to convey information from mind to mind. His problem is less one of linguistics than one of logical thinking and awareness of the person to whom the information is being conveyed. A well thought out structure, coherence, means for leading the readers attention easily from one point to the next, a sense of what is and what is not relevant, are the most essential qualities. This is appreciated by the author of the book under review, who has had many years experience in teaching the subject. He brings to the book the ripe fruit of that experience.—R. O. KAPP.

THE WORLD'S OILFIELDS. *The Eastern Hemisphere. Science of Petroleum. Vol. VI, Part I. Edited by V. C. Illing. Oxford University Press, London 1953. 180 pp. 52s. 6d. net.*

This volume brings up to date that portion of Volume I of "The Science of Petroleum" which dealt with the occurrence of Petroleum in the Eastern Hemisphere. The Editors have had a difficult task and have succeeded admirably in co-ordinating the subject matter of over thirty articles so that the book presents a more integrated study of the whole Eastern Hemisphere than was achieved in the earlier volume. The contents are divided into four main sections, dealing respectively with Europe, The Middle East, The Far East and Australasia and Africa. Within each section separate chapters describe in detail the geological features of each producing area. Unfortunately, areas within the U.S.S.R. have had to be omitted because of lack of current information, and the chapters dealing with other areas within the U.S.S.R. satellite countries are admittedly based on information now possibly out of date. With these reservations the book provides an excellent and up to date account of the Oilfields of the Eastern Hemisphere.

The general pattern of presentation followed throughout the text is as follows: A brief introduction and historical note is followed by details of the stratigraphy, tectonics and geophysics of the area. The oil source rocks are then discussed followed by detailed classification of the oil deposits. In areas where commercial deposits occur in a variety of zones these are discussed separately. Much use is made of tabular presentation and the text is well illustrated by maps and diagrams. Naturally in such a wide survey certain articles are of greater value than others. From the student's view point the articles on the Middle East, on Germany, France and Great Britain are of considerable interest and extremely well presented. Those on Italy, Switzerland and Yugoslavia are disappointing and of little value.

It is impossible to review such a text in any detail, since much of the information is of a specialist nature. The book is of the utmost importance to students and workers in the field of petroleum geology but it is unfortunate that publication costs should place the book—and the whole series of the Science of Petroleum—at such a price that it is beyond the means of the student for whom it has the greatest value.—F.M.

JANE'S ALL THE WORLD'S AIRCRAFT 1952-53. *Edited by Leonard Bridgman. Sampson Low, London 1952. £4 4s.*

This forty-third issue of "Jane's" is a little easier to review than its predecessors, because it is somewhat different in arrangement; and this may be the time to suggest other and minor changes that might be made in the future. But first of all the traditional, and well deserved, praise must go to Mr. Bridgman for his excellent editing and arrangement: the standard he preserves is now a byword, and it is through no fault of his that the first criticism must be of the reduction in the overall size of the new edition. It is great pity that through advertising, or even by a slight increase in price, the previous size could not have been preserved.

To retain as much material as possible on the aircraft themselves, and the engines, the military section has been reduced to merely the air force markings of the world—a wise decision, as few nations are willing to release the true state of their service aviation. The Civil Aviation section now includes only notes on I.C.A.O., the F.A.I., civil aircraft markings, records, and a list of the world's air lines with addresses and notes on the fleets operated.

The main sections, Parts C and D, which are the main *raison d'etre* of "Jane's," include 573 illustrations of aircraft, of which 311 are new. One would have liked to see these increased, and also to see more of Mr. L. E. Bradford's admirable line drawings, and any reviewer must cast an envious eye at the 101 pages of advertisements on art paper.

The overall reduction must, of course, be our first point of criticism. Could not the advertisements, or the bulk of them, be printed on less luxurious paper? and could not they be placed *behind Jane's All the World's Aircraft* instead of in front of this great work? Most of us use "Jane's" on a table, and to have to go more than a quarter of the way through the volume before arriving at the reference material is more than a little irritating.

The present reviewer would also like to see a longer analysis by Mr. Bridgman of trends and events, more general aviation reference material, such as addresses of aero clubs, statistics about aircraft production and air line operation, and so on. Such material, a great deal of it, could be included on cheaper paper, and form an invaluable collection.

However, nothing can take the place of "Jane's," however shorn it may appear, and long may it flourish.—C. H. GIBBS-SMITH.

BRACKLES. MEMOIRS OF A PIONEER OF CIVIL AVIATION. *Frida H. Brackley. W. & J. Mackay & Co. Ltd. 1952. 695 pp. Photographs. 25s. net.*

As one of Brackles many admirers, and an old friend who flew with him in the early days and on, I am proud to pay tribute to the story of his life, compiled with loving thought by a devoted family.

Rather than analyse the book it will be my aim to stress the significance of his pioneering in the technological and operational fields. It is well that we should remind ourselves, while the memory of a vivid and most engaging personality to which Air Transport owes so much, is yet fresh in our minds.

The Elizabethans have their counterpart in this Twentieth century, and with these few Brackles surely takes his place. How well the lines of Charles Sprague fit him:

"In the fields of the air he writes his name,
And treads the chambers of the sky."

The first to conceive the fearsome idea of "bombs by the ton" was Sir Murray Sueter, in charge of the R.N.A.S. in the first War. The Admiralty asked Sir Frederick Handley-Page to design the first ever twin-engined heavy Bomber. It carried a ton of bombs. Brackles, a Squadron Commander, was in the lead in piloting one of these through the flak to Zeebrugge and other targets.

He had developed to a remarkable extent all the natural aptitudes that, combined together, make the best pilot. He was at his happiest when handling a large Flying Boat, and played a leading part in developing and operating the Empire Class of Short boat to the design of Sir Arthur Gouge.

In 1921, Brackles joined me in the British Aviation Mission to the Imperial Japanese Navy. During the years in that service he increased his experience of Flying Boat operations under a wide variety of climatic conditions, and made the first deck landing with an amphibious Flying Boat on the I.H.N. Carrier Hosho.

So long as he held a leading position in B.O.A.C., or its sister B.S.A.A., those of us who were keenly watching Flying Boat development, and appreciated its importance, knew that full advantage would be taken of its potential, and encouragement given to the development of new types. The loss of Brackles has severed this vital link. The catastrophic effect of this loss can be seen today since The Princess, the world's premier ocean going Flying Boat, the latest design of Sir Arthur Gouge developed out of the experience gained when Empire air communications were exclusively carried by Flying Boat, is with one exception shore-bound.

The first of these, under-engined though she is, is carrying out useful development trials. The other two, part finished, have been committed to a plastic grave where no doubt moth and rust will not corrupt, nor, thanks to the vigilance of the Saunders-Roe Watchmen, will thieves break in and steal.

This officially decreed stagnation is, mildly expressed, a mark of ingratitude to the memory of a man who played so notable a part in developing Imperial Air communications by Flying Boat.

Brackles was very dear to me, and I can look back with happiness to a day in the Land Of The Rising Sun when in the shadow of the great Kwannon at Kamakura I introduced him to Freda Mond. All who knew Brackles, will want to add to their knowledge by reading her account of a life packed with achievement.—SEMPILL.

THE HIGH SPEED INTERNAL COMBUSTION ENGINE. *Sir Harry R. Ricardo. Blackie & Son Ltd. 1953. Fourth edition. 420 pp. 226 figures. 40s. net.*

Ricardo's book on "The High Speed Internal Combustion Engine," first published by Messrs. Blackie in 1923, now appears in its fourth edition entirely rewritten and reset. The last revision, it will be remembered, was to a considerable extent the labours of the late Mr. H. S. Glyde, one of Ricardo's outstanding assistants.

Sir Harry, the indefatigable, has now made a fresh start and given a most able exposition of the subject, by condensing into one volume the activities in research and development at his firm's engine laboratory over the past 35 years.

The book is divided into 19 chapters, the first five of which deal principally with the fuel aspect as affecting engine behaviour. The next two chapters are on combustion chamber design with the emphasis on the C.I. engine. Chapters on cold starting of the C.I. engine, mechanical efficiency, supercharging and the two-stroke follow. The author then introduces the subject of mechanical design (two chapters). The important problems

of the piston, piston cooling, and cylinder wear are next covered. There is a well illustrated chapter on piston type aero engines followed by a full account of sleeve valve engines. A very interesting chapter deals with the author's work on two-stroke sleeve valves, though he admits there is little present evidence of the type. The final chapter concerns engines for research.

The work of the Ricardo Laboratory at Shoreham-by-Sea, and at Oxford during the Second World War, has pioneered the way of the high speed internal combustion engine from the days of the first army tank engine of the First World War up to the present time. Ricardo has been personally responsible for many new design concepts and research and development projects. He has achieved such clarification of ideas and principles that the most important technical advances have sprung from them. It is not an exaggeration to add that the universal use of the automotive engine has come about more quickly as the result. The personal charm of the leader has added much to the intrinsic worth of the laboratory investigations. This same delightful manner distinguishes his written work.

In the early days of the automotive engine only the petrol or spark ignition engine existed. Except in the smaller sizes, and in the pleasure car field, where because of first cost it still remains supreme, the petrol engine today is all but superseded by the compression ignition engine. His work towards the development of the compression ignition engine is one of the most outstanding of Ricardo's achievements.

It is a tremendous and still widening field which says much for the continued vigour of the piston type engine. The writer well remembers Ricardo's work on the gas turbine in the Second World War, and one would have thought it a golden opportunity to foresake the piston engine for the intriguing new development of the gas turbine. But Ricardo with few regrets left gas turbine research to get back, as soon as possible, to the more solid work of the piston engine.

It says a great deal for the type of work which the author describes that it was fostered almost continuously by the Air Ministry or Research Directorates of the Ministry of Supply and was often done in collaboration with the main aero engine firms. Some of the story, for example the early progress of the C.I. aircraft engine, makes fascinating reading. Credit for the first demonstration that the high speed engine would work on compression ignition rests with the late H. B. Taylor of the Royal Aircraft Establishment and this was a discovery of great importance. Although C.I. could not keep pace with the petrol engine in the race for greater and greater outputs during 1939-1945, because it could not benefit from detonation free fuels, today with the aircraft engine almost entirely gas turbine, the C.I. engine holds dominate sway in the piston engine field.

Another development which is described and which will long be considered predominently the pioneering work of Ricardo, is the Burt-McCollum sleeve. Ricardo, as he describes himself, is a middleman between the extremes of out-and-out scientist and the entirely practical engineer, and he was attracted by the sleeve valve as a practical mechanism with great theoretical advantages. Introduced on the results of Shoreham Laboratory work, this became the standard valve for aero engines built by Bristol and Napier and it was also used in the Eagle, the last Rolls-Royce piston type aero engine.

Another of Ricardo's activities has been long concerned with supercharging or boosting and its effect on performance and design. Between the wars a legend grew that the highest boosts and highest B.M.E.P.'s were always reached first at Shoreham. The highly boosted C.I. engine, which worked up to two atmospheres boost or more, set forth a whole series of developments using both the Ricardo sleeve valve and the Ricardo Comet type head. This work showed the possibilities of supercharging a two-stroke engine by applying an exhaust back pressure in order to retain charge in the cylinder; which led on to the utilisation of the exhaust energy in a turbo supercharger and foreshadowed the type of cycle used in the composite engine.

In his final chapter on Engines for Research, the author mentions his early researches under Hopkinson and of the outstanding work with Tizard and Pye which enabled fuels to be classified in terms of knock rating for the first time.

From a lifetime of experience he draws morals and conclusions which show that research to improve the high speed internal combustion engine is a prolonged and costly process in which time can only be saved by sound judgement aided by intelligent analysis of carefully conducted experiments.—A. W. MORLEY.

DESIGNING BY PHOTOELASTICITY. *R. B. Heywood. Chapman & Hall, London 1952. 414 pp. 65s. net.*

Dr. Heywood has addressed his book both to photoelasticians and engineers. It falls naturally into two parts: Part I, which occupies approximately one-third of the length of the book, deals with some of the fundamentals of optics which underlie the use of the photoelastic method of stress analysis; with the properties of photoelastic model materials, the preparation of models and the necessary precautions for accurate loading of models; with the validity of the transfer of information gained from a model made of a transparent plastic to a prototype of mild steel or light alloy; and with the fundamentals of the " frozen stress " technique used in the analysis of three-dimensional stress problems, together with a considerable amount of practical information.

Part II is devoted to the discussion of information derived from photoelastic data, as well as from theoretical and empirical work by the author and others on stress concentrations caused by notches and holes. One chapter is devoted to stress concentrations in screw threads, bolts and nuts, while another discusses some improvements component design obtained by paying attention to geometrical details such as fillets, after the original designs had been found wanting by photoelastic analysis. The final chapter of the book discusses the application of stress concentration factors, their relation to problems where fatigue is an important consideration and the effects of size on the applicability of these factors. The book is well illustrated and is completed by an extensive bibliography.

Part I seems on the whole to have been designed for the reader who wants information on practical engineering problems; it is therefore surprising that the author should have included in this part a discussion of fringe-strain as distinct from fringe-stress coefficients, which involves a number of assumptions which make the analysis of doubtful value. The author is also very dogmatic about the relative merits of the diffuse light polariscope and the polariscope which employs a small light source. Although this reviewer agrees with some of the conclusions, it seems unnecessary to him to characterise the more accurate polariscope merely as " impressive show pieces." Similarly sweeping statements are made which may mislead readers unfamiliar with this subject; for example, that only boundary stresses are of real interest, and it is implied that

methods dealing with whole regions should be left to those who have nothing better to do.

These dogmatic asides which occur in various places throughout the book should not, however, detract from the credit due to the author for collecting within one book a great deal of information about stress concentrations which was previously scattered. This information should be useful to designers and engineers with sufficient background knowledge of the subject matter.—E. K. FRANKL.

ELASTICITY IN ENGINEERING. *E. E. Sechler. Chapman and Hall, 1952. 419 pp. Diagrams. 68s.*

The modern engineer is a lucky fellow as several people have, in recent years, eased his burden by writing books which clarify the mathematical theory of elasticity. The present book is another attempt in this direction and, as stated by the author, has been written primarily for engineers working in all fields of structural analysis.

The book is divided into three parts. The first part recites the fundamentals of the theory of elasticity and takes the reader, by easy stages, through stress and strain, stress-strain relationships, compatibility, the stress function and elastic energy principles. Work in three dimensions is not neglected and the appropriate equations in various co-ordinate systems are clearly discussed. Although there is nothing new in the subject matter, this part of the book is valuable as it expounds the various topics with detail and clarity.

The second part of the book deals with engineering problems in stable structures. The first chapter discusses two-dimensional stress problems in Cartesian and cylindrical co-ordinates; after the earlier reading this chapter appears to be unduly condensed and somewhat disjointed. The next chapter is solely concerned with the theory of bending and is an excellent exposition of this subject. The following chapter on continuous and redundant structures comes as quite a shock as the reader has now to switch his attention to methods of structural analysis. This is again too condensed and attempts to cover too wide a field in travelling from redundant structures to beams on elastic foundations. Illogically, the next chapter returns to the theory of elasticity with an excellent description of the torsion problem; the differences between Saint Venant's method, the stress function and energy solutions being clearly defined. A mild criticism is that, particularly for aeronautical engineers, a little more emphasis on warping should have been given. The final chapter in this part of the book is devoted to plates and shells. There is nothing fresh here and the work is so condensed that the reader might just as well look at Timoshenko's *Theory of Plates and Shells*, which is one of the references.

The final part of the book attempts to describe, in some fifty pages, engineering problems in stability. Although well written it attempts too much in dealing with columns, beam columns and the instability of plates and shells.

The overall impression which this book gives is that it covers too wide a field; it is, however, written in a very easy style and well illustrated. Problems are given at the end of the chapters but unfortunately answers are not provided; references for collateral reading are included. The publishers are to be congratulated on the beautiful production of the book.—S. C. REDSHAW.

ELEMENTS OF PROPELLER AND HELICOPTER AERODYNAMICS. *D. O. Dommasch. 178 pp. Pitman. 35s. net.*

The author points out in his Preface that, " for the most part, helicopter aerodynamics is merely an extension of propeller theory " and considers it logical to present the elements of the two subjects in one course of lectures. These lectures, prepared for the students of Iowa State College and for test pilots of the U.S. Naval Air Test Centre, are now presented in book form for the benefit of other students.

In the first three chapters the author reviews the basic principles of aerodynamics and the momentum and blade element theories of the propeller. To summarise the principles of aerodynamics in twenty pages is not an easy task and the author spoils his effort by wasting space on an original and spurious " derivation " of the Kutta-Joukowski law relating lift to circulation and by giving diagrams and formulae relating to supersonic drag which are not referred to elsewhere in the book. The propeller theories of Glauert, Goldstein, Theodorsen and Lock are briefly described and examples are worked in the text to illustrate the methods. Each chapter ends with a summary, a list of references and a set of numerical problems for the pupil.

The last three chapters are devoted to the helicopter—in vertical flight, in horizontal flight and to its stability and control. The equations for rotor thrust and torque in vertical flight are merely adaptations of the propeller equations and perhaps for this reason, the numerical examples are worked in terms of $J(=V/\{nD\})$ and $C_T(=T/\{\rho n^2 D^4\})$ symbols which are not customarily used in helicopter aerodynamics. In the chapter on forward flight the continued use of r.p.m. instead of the usual angular velocity leads to equations involving π^4 so that the helicopter's ancestry in propeller theory becomess an embarrassment. Few helicopter workers will find it difficult to adopt a new set of symbols more convenient to helicopter analysis. In the final chapter on control and stability the method of equation and numerical solution has been abandoned and the physical nature of this difficult subject well described.

This book will be a help to the student who has studied propeller theory and who now wants to apply the calculations to the helicopter. With its aid he will be able to work helicopter performance problems but will need much more study to realise the significance of the many assumptions made in these preliminary lectures.—A. H. YATES.

POWER PLANTS FOR AIRCRAFT. *By Joseph Liston McGraw-Hill. 577 pp. Diagrams. 72s. 6d.*

The title of this book may be misleading to English readers, as it deals with aircraft engines and not with power plants as understood here. Installation problems and the relation of engines to aircraft performance are hardly discussed at all.

Chapter I, " Basic Requirements and Achievements," is extremely sketchy. The requirements of propulsion demanded by the aircraft, a subject which has been extensively studied in recent years, are not analysed at all. Such studies, being readily treated by elementary mathematics, would have formed an admirable guide to such a subject. Instead a curve is shown illustrating the field of use of various engines without any reasons being given. In describing the basic historic types of reciprocating engine the reader will be interested so see from the table on page 10 that from 1901 to 1930 no British engine made any marked contribution to progress in Aeronautics, and only two of non-American design are mentioned!!

Chapter II, " Utilisation of Available Energy," 64 pages, deals first with airscrews and then with the various types of propulsion unit in a very elementary fashion, indicating the basic principles of each.

Chapter III on " Fuels " (70 pages) and Chapter V on " Cylinder and Burner Phenomena " (100 pages) are by

contrast quite full accounts of the subject, considering the general scope of the book, and together would appear to indicate where the author's real interest lies. These two chapters are to be recommended, but surely the inclusion of eight pages on nuclear fuels is both out of place and shows lack of proportion?

Chapter IV on " Charge Handling " (86 pages) describes fuel systems, carburettors, air intakes and compressors. The inclusion in one chapter of systems for both reciprocating and turbine engines is not in the reviewer's opinion a happy one. One would have hoped to find here also a clear account of the calculation of the air flow capacity of a piston engine, but neither here nor later is there any attempt at a theoretical approach to this susbject which is so necessary when supercharging is considered.

Chapter VI on " Performance " (83 pages), gives a very good account of the methods of measurement of torque, fuel and air consumption, speed, pressures etc. and various designs of pressure and temperature probes used for detailed exploration. The section on altitude performance and correction suffers from lack of a theoretical treatment of the power output of piston engines mentioned earlier.

Chapter VII on " Turbine Engine Characteristics " (60 pages) deals with the various cycles and cycle calculations and is quite adequate. It contains a great number of useful charts for the student.

Chapter VIII on " Structural and Mechanical Requirements " (89 pages) deals first with cylinder cooling in some detail but, together with the section on cowling, shows that the author is thinking of air-cooled engines and has forgotten liquid-cooled types. The other sections dealing with mechanical design are adequate considering the general scope of the book, but that on turbine engines is sketchy and appears to be a mere appendage to this chapter.

A very uneven book. The author states that it is intended for senior and beginning graduate students; the expenditure of so much money by any student in this country could hardly be recommended by this this reviewer.—H. P.

THERMAL DIFFUSION OF GASES. *K. E. Grew and T. L. Ibbs. Cambridge University Press 1952. 143 pp. 40 figures. 22s. 6d. net.*

" Thermal Diffusion " is the term applied to the relative motion of components of a mixture caused by differences of temperature within it. Such a diffusion process may be used to bring about partial separation of the components of an initially uniform mixture and has in fact been applied in separation of gaseous isotopes of a number of elements.

This monograph reviews the present state of knowledge of the subject in a clear and concise manner. In the early chapters the phenomenon and the experimental methods used to observe it are described, and the magnitude of the effect is compared with that predicted by simple theory. It is of interest to note that the effect itself was observed in gases only after it had been predicted from theoretical considerations. The related " Diffusion Thermo-effect "—temperature change as a result of diffusion of one gas into another—is also described.

Chapters VII and VIII deal briefly with the practical application of thermal diffusion to the separation of gaseous mixtures, and the thermal diffusion effect in liquids.

Too many years ago, as a "fresher" engineering student, the reviewer found he was to take what to him seemed an unnecessary Physics course. He is still not convinced that the course was necessary, but Dr. Ibb's gift of exposition made it quite painless. Grew and Ibbs have brought the same clarity of expression to this monograph. In general, the text is free from all but essential theory. Essential theoretical results are quoted and explained, the detailed theory being arranged in a series of Appendices. The main concern is with experimental methods and results, while for any who wish to delve more deeply into the subject, there is a comprehensive bibliography.

For the engineer this is a readable little book on a subject which is not without topical interest; for the physicist it fulfils the useful task of reviewing and correlating the considerable mass of experimental information on this subject.—H. BILLETT, University College, London.

TRANSPORT BY AIR. *Thomas Insull. Designed and edited by Paul Redmayne. John Murray, London 1953. 48 pp. Illustrations. 9s. 6d. net.*

This is the sixth book in " The Changing Shape of Things " series. Others have dealt with transport by land, transport by sea, houses, dress and things in general that have changed their shape. It is described as a Graphic History of Flight and is one of the most compact works of its kind. There are twenty-two sections, the majority of them lavishly illustrated, compressed into a slim volume of forty-eight pages.

It is not the type of book that most members of the Royal Aeronautical Society stand in desperate need of, but they might find it a useful defence against a too-curious air-minded nephew, or any other aeronautically-inclined, fact-seeking youngster, or ignorant grown-up. Let the author state his own case. " This book," he says in the foreword, " is about the history of flight from its earliest days. It is not written in technical language, nor has it been compiled by technical experts, because it is less concerned with the scientific aspects of aeronautics than with the influences which have brought about changes in the design of aircraft, and with the chief results of mechanical flight."

The author has discharged his obligation to his readers with commendable skill and economy of words. The hypercritical might find a fault here and there, but the generous-minded will concede that the performance accords reasonably well with the specification. Nevertheless, no air line executive with sad memories of fog-bound airports all over Europe last winter is likely to endorse the sweeping statement on page 22 that: " By 1918 airmen could pilot an aircraft in darkness at great heights and in all weathers and climates." Or can it be that we have slipped a little in the past thirty-five years?—S.E.V.

Additions to the Library

Davis, D. S. EMPIRICAL EQUATIONS AND NOMOGRAPHY. McGraw-Hill. 1943.

Guggenheim Medal Board of Award. PIONEERING IN AERONAUTICS. (Recipients of the Guggenheim Medal.) 1953.

Kamminga, M. S. THE AIRCRAFT COMMANDER IN COMMERCIAL AIR TRANSPORTATION. Nijhoff, the Hague. 1953.

Meer, H. P. van, and F. J. Plantema. FATIGUE OF STRUCTURES AND STRUCTURAL COMPONENTS. TIBT/4080. Ministry of Supply translation of N.L.L. Report. 1949.

Admiral Lord Mountevans. ARCTIC SOLITUDES. Lutterworth. 1953.

S.B.A.C. STANDARD METHOD FOR THE ESTIMATION OF DIRECT OPERATING COSTS OF AIRCRAFT. 1953.

Reports

AERODYNAMICS

Boundary Layer

See also Wings and Aerofoils

The two-dimensional laminar boundary layer at hypersonic speeds. T. Nonweiler. Cranfield Report No. 67.

A numerical solution is found for the equations governing the motion of a two-dimensional laminar boundary layer, in the absence of a pressure gradient, which would be valid if the flight Mach number is very high (*i.e.* $M^2 >> 1$). The effects of surface slip, and the finite thickness of the boundary layer are shown to be negligible if the Reynolds number (R) exceeds about 10^5, and are neglected.—(1.1.1 × 1.2.3).

The theoretical estimation of power requirements for slot-suction aerofoils, with numerical results for two thick Griffith type sections. J. H. Preston, N. Gregory and A. G. Rawcliffe. R. & M. 2577.

This report describes a method for assessing the performance of slot-suction aerofoils in terms of an effective drag coefficient, which takes into account the power requirements of the suction pump neglecting slot entry and duct losses. When the suction-slot is located at a velocity discontinuity the suction flow required to prevent separation can be calculated, using the elementary theory suggested by Sir Geoffrey Taylor.—(1.1.5).

Effect of thermal properties on laminar-boundary-layer characteristics. E. B. Klunker and F. E. McLean. N.A.C.A. Technical Note 2916.

An iteration method is presented for solving the laminar-boundary-layer equations for compressible flow in the absence of a pressure gradient wherein the temperature variation of all the fluid thermal properties is considered. Friction and heat-transfer characteristics have been calculated for a stream temperature of −67° F for Mach numbers from 1 to 10 with the use of values of the heat capacity, conductivity, and viscosity determined from experiment.—(1.1.1).

A modified Reynolds analogy for the compressible turbulent boundary layer on a flat plate. M. W. Rubesin. N.A.C.A. Technical Note 2917.

A modified Reynolds analogy is developed for the compressible turbulent boundary layer on a flat plate.—(1.1.3).

Control Surfaces

The effect of curvature of surface and thickness of trailing edge on aileron hinge moments. A. S. Batson, et al. R. & M. 2506.—(1.3.1).

Fluid Dynamics

Tables of complete downwash due to a rectangular vortex. Computed by the staff of the mathematics division, N.P.L. R. & M. 2461.

Critical tables are given from which the complete downwash due to a unit rectangular vortex can be read to three place of decimals, on selected lines coplanar with the vortex. These lines are at specified integral multiples of the semi-width of the vortex from the centre line of the vortex, and are of importance in the solution of wing loading problems by vortex lattice theory.—(1.4).

Impingement of water droplets on a cylinder in an incompressible flow field and evaluation of rotating multicylinder method for measurement of droplet-size distribution, volume-median droplet size, and liquid-water content in clouds. R. J. Brun and H. W. Mergler. N.A.C.A. Technical Note 2904.

The trajectories of water droplets in an incompressible flow field around a cylinder were calculated with a mechanical analog. The collection efficiency, the area of droplet impingement on the cylinder, and the rate of droplet impingement were determined from the trajectories.—(1.4).

On the development of turbulent wakes from vortex streets. A. Roshko. N.A.C.A. Technical Note 2913.

Wake development behind circular cylinders at Reynolds numbers from 40 to 10,000 was investigated by hot-wire techniques in a low-speed wind tunnel.—(1.4).

Internal Flow

The asymmetric adjustable supersonic nozzle for wind-tunnel application. H. J. Allen. N.A.C.A. Technical Note 2919.

An asymmetric adjustable nozzle for supersonic wind-tunnel application which permits continuous adjustment of the test-section Mach number is described. The characteristics of this nozzle are compared with the more conventional supersonic tunnel nozzles.—(1.5.1).

The design of variable Mach number asymmetric supersonic nozzles by two procedures employing inclined and curved sonic lines. C. A. Syvertson and R. C. Savin. N.A.C.A. Technical Note 2922.

Two procedures are developed for designing asymmetric supersonic nozzles for which the calculated exit flow is essentially uniform over a range of Mach numbers. One procedure is applicable at Mach numbers below approximately 3; the other procedure is used for designs at Mach numbers exceeding 3.—(1.5.1).

Wings and Aerofoils

Note on research into some aspects of stall warning devices. C. F. Bethwaite and Lt. R. A. Langley. Cranfield Report No. 72.

The problems of detecting and indicating an approaching stall have been investigated in flight on an "Anson" aircraft. A lower-surface flap near the leading-edge of the wing detects the approaching stall at a speed which depends critically on the length of the flap and on its location, but the margin of warning speed over stalling-speed is reasonably independent of landing-flap position and throttle setting.—(1.10.2.2).

Aerodynamic forces on biconvex aerofoils oscillating in a supersonic airstream. W. P. Jones and Sylvia W. Skan. R. & M. 2749.

A method for the calculation of the aerodynamic forces on an oscillating aerofoil which allows for the effect of thickness is developed. The steady flow régime for zero or mean incidence is first determined for isentropic, irrotational and inviscid flow conditions. A small disturbance is then assumed and the non-linear equation defining the subsequent motion is reduced to a linear equation for the velocity potential of the disturbance. This is expressed in difference form and solved numerically by a step-by-step process.—(1.10.1).

Détermination du point de transition et calcul de la trainée des profils d'ailes en incompressible. R. Michel. O.N.E.R.A. France. Publication No. 58.—(1.10.2.1 × 1.1.2).

On the calculation of the pressure distribution on three-dimensional wings at zero incidence in incompressible flow. O. Holme and F. Hjelte. KTH, Sweden, Aero T.N. 23.

A short account is given of the linearised theory of three-dimensional wings at zero incidence in incompressible flow. The theory is applied to two special cases, where the perturbation velocity may be expressed in terms of elementary functions, namely a rectangular wing with parabolic arc section, but with a relative thickness decreasing linearly towards the wing tips.—(1.10.1.2).

Aerodynamic forces on oscillating low aspect ratio wings in compressible flow. H. Merbt and M. Landahl. KTH, Sweden, Aero T.N. 30.

In the case of pointed wings of very low aspect ratio, the x-derivatives can be neglected in the linearised potential equation, which thus reduces to the two-dimensional wave equation. The resulting flow, being two-dimensional in cross-sections perpendicular to the free stream direction, is calculated in terms of Mathieu functions.—(1.10.1.2).

NOTE:—*The figures in parenthesis at the end of each Summary are for office use only.*

HELICOPTER AERODYNAMICS

The normal component of the induced velocity in the vicinity of a lifting rotor and some examples of its application. W. Castles, Jr. and J. H. De Leeuw. N.A.C.A. Technical Note 2912.

A method is presented for computing the approximate values of the normal component of the induced velocity at points in the flow field of a lifting rotor. Tables and graphs of the relative magnitudes of the normal component of the induced velocity are given for selected points in the longitudinal plane of symmetry of the rotor and on the lateral rotor axis. A method is also presented for using the tables and graphs to determine the interference induced velocities arising from the second rotor of a tandem- or side-by-side-rotor helicopter and the induced flow angle at a horizontal tailplane.—(1.11).

AEROELASTICITY

Essai de vibrations au sol d'une maquette d'avion. J. Rost. O.N.E.R.A. France. Note Technique No. 13.—(2).

Construction de maquettes dynamiquement semblables destinées a l'étude du flutter en soufflerie. R. Melzer. O.N.E.R.A. France. Publication No. 59.—(2).

ELECTRONICS

Transducteurs et amplificateurs magnétiques. K. Kuhnert et M. Delattre. O.N.E.R.A. France. Publication No. 57.—(11).

MATERIALS

A linear time-temperature relation for extrapolation of creep and stress-rupture data. S. S. Manson and A. M. Haferd. N.A.C.A. Technical Note 2890.

A time temperature parameter based on examination of the published stress-rupture data for a variety of materials, rather than on a physical theory, is proposed in terms of temperature, rupture time and material constants determinable from rupture data in the time range between 30 and 300 hours. Extrapolations to longer times up to 10,000 hours were also investigated.—(21.2.1 × 21.2.2 × 33.2.2).

MATHEMATICS

Problèmes elliptiques et hyperboliques singuliers pour une équation du type mixte. P. Germain et R. Bader. O.N.E.R.A. France. Publication No. 60.—(22.1).

Le point de vue du physicien dans le problème du lissage des courbes expérimentales. P. Vernotte. Publications Scientifiques et Techniques du Ministère de L'Air. No. N.T. 46.—(22.1).

The solution of eigenvalue problems of integral equations by power series. J. R. M. Radok. A.R.L. Australia. Report SM. 201.

The characteristic equations and eigensolutions of Fredholm integral equations of the second kind with geneous polynomial type are obtained by use of power series. The method is applied to the particular case of the kernel $(x\text{-}y)$ and it is indicated that it may readily be extended to more general types of equations.—(22.2).

MECHANICAL ENGINEERING

Comparison of high-speed operating characteristics of size 215 *cylindrical-roller bearings as determined in turbojet engine and in laboratory test rig. E. F. Macks and Z. N. Nemeth. N.A.C.A. Report* 1084.

Inner- and outer-race cooling-correlation curves were obtained for the turbo-jet engine turbine-roller bearing with the same inner- and outer-race cooling-correlation parameters and exponents as those determined for the test-rig bearing. The inner- and outer-race turbine roller-bearing temperatures may be predicted from a single curve regardless of variations in speed, load, oil flow, oil inlet temperatures, oil inlet viscosity, oil-jet diameter, or any combination of these parameters. This report was formerly designated as RM E51I05.—(23.1 × 27.1).

POWER PLANTS

See also MECHANICAL ENGINEERING

Régulation hydraulique d'alimentation des turbomachines. B. Demtchenko. Publications Scientifiques et Techniques du Ministère de L'Air. No. 277.—(27.1).

PROPELLERS

Sound from a two-blade propeller at supersonic tip speeds. H. H. Hubbard and L. W. Lassiter. N.A.C.A. Report 1079.

Propeller sound measurements at static conditions have been extended to a tip Mach number of 1·30. Spectrums have been obtained at both subsonic and supersonic tip speeds for comparison, and the measured data are compared with calculations by the Gutin theory. At supersonic tip speeds, the maximum intensities were measured in the plane of rotation and the spectrums are noted to contain a large high-frequency content. This report was formerly designated RM L51C27.—(29.1 × 5.3).

Study of the supersonic propeller. J. Fabri and R. Siestrunck. N.A.C.A. Technical Memorandum 1355.

In this paper a propeller having all sections operating at supersonic speeds is designated a supersonic propeller regardless of flight speed. Analyses assume subsonic flight speeds but very high rotational speeds. A very elementary analysis of the efficiency of a jet-propeller system is presented. Use is made of theoretical aerofoil characteristics at supersonic speeds. A study of blade section interference, blade shock and expansion fields, at supersonic section speeds is presented.—(29.1).

RESEARCH

Effects of parallel-jet mixing on downstream Mach number and stagnation pressure with application to engine testing in supersonic tunnels. H. Bernstein. N.A.C.A. Technical Note 2918.

A one-dimensional analysis of the results of the parallel-jet mixing encountered in the testing of engines in supersonic wind tunnels is reported. This type of analysis presents a reasonable approach to obtaining approximate figures for the tunnel operating conditions while the tunnel is still in the design stage.—(31.2).

SCIENCE—GENERAL

Contribution a l'étude de l'intensité des bandes d'absorption infrarouge. G. Amat. Publications Scientifiques et Techniques du Ministere de L'Air. No. 276.—(32.2.4).

STRUCTURES

See also MATERIALS

Direct-reading design charts for 24*S-T3 aluminium-alloy flat compression panels having longitudinal formed hat-section stiffeners and comparisons with panels having Z-section stiffeners. W. A. Hickman and N. F. Dow. N.A.C.A. Technical Note* 2792.

Direct-reading design charts are presented for the determination of the stress and proportions of 24S-T3 aluminium-alloy flat compression panels having longitudinal formed hat-section stiffeners, for given values of intensity of loading, skin thickness, and effective length. Hat- and Z-stiffened panels are compared as compression panels and as the covers of box beams.—(33.2.4).

Inelastic column behavior. J. E. Duberg and T. W. Wilder, III. N.A.C.A. Report 1072.

This paper presents the significant findings of a theoretical study of column behaviour in the plastic stress range that shows the tangent-modulus load to be the load at which bending can start for a straight column. Load-deflection analyses are made for two column models—one model having a concentrated flexibility and the other having its flexibility distributed along its length. Results are presented which relate the maximum load to the stress-strain curve of the material.—(33.2.4 × 33.2.2).

APPOINTMENTS

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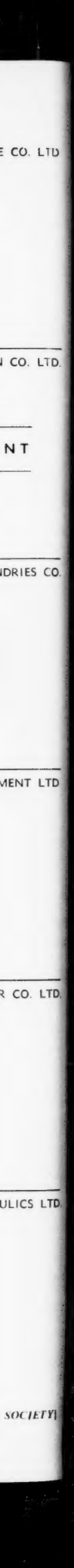
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